

PG - II

PAPER - EMT & ELECTRODYNCS

UNIT - VI (BASIC IDEA OF
PLASMA CONFINEMENT)

PART - I, II, III

References -

1. Introduction to Plasma Physics
- F. F. Chen
2. Introduction to Plasma Physics
- Goldston & Rutherford
3. Fundamentals of Plasma Physics
- J. A. Bittencourt.

Supervised by - ARUP BHARALI

PART - I

Motion of charged particles (e^- & ions in plasma) in
CONSTANT and UNIFORM \vec{E} and \vec{B} fields.

1
4/V

- The eqn of motion for a particle of charge q under the action of the Lorentz force \vec{F} due to electric (E) and magnetic induction (B) fields, can be written as -

$$\boxed{m \frac{d\vec{v}}{dt} = \vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]} \quad \begin{array}{l} \text{(Non-relativistically)} \\ \rightarrow ① \end{array}$$

- Taking the dot product of ① with \vec{v} →

$$m \vec{v} \cdot \frac{d\vec{v}}{dt} = q[(\vec{v} \cdot \vec{E}) + \vec{v}_0 (\vec{v} \times \vec{B})]$$

$$\Rightarrow \frac{d}{dt} \left(\frac{1}{2} m v^2 \right) = q \vec{v} \cdot \vec{E} = 0.$$

$$= q \vec{v} \cdot (-\vec{\nabla} \phi),$$

$$= -q (\vec{\nabla} \phi) \cdot \frac{d\vec{r}}{dt}$$

$$= -q \frac{d\phi(\vec{r})}{dt}$$

$$\Rightarrow \frac{d}{dt} \left[\frac{1}{2} m v^2 + q\phi \right] = 0.$$

$$\vec{F} = \frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt}$$

m → mass

$$\phi = \phi(\vec{r})$$

$\phi \rightarrow$ Electrostatic potential
energy per unit charge

$$\vec{\nabla} \phi \cdot \frac{d\vec{r}}{dt}$$

$$= \frac{\partial \phi}{\partial x} \frac{dx}{dt} + \frac{\partial \phi}{\partial y} \frac{dy}{dt} + \frac{\partial \phi}{\partial z} \frac{dz}{dt}$$

$$\Rightarrow \frac{d\phi}{dt}(x, y, z) \quad \underline{\text{Chain Rule}}$$

i.e. the total KE + electric potential energy of a charged particle moving in a constant/static E-M field is a constant of motion, i.e.

$$\boxed{\frac{1}{2} m v^2 + q\phi = \text{a constant of motion}} \rightarrow ②$$

There is no role of magnetic field in the conservation principle as long as static & uniform magnetic field is concerned. For presence of only magnetic field (uniform and constant) \rightarrow $\boxed{\frac{1}{2} m v^2 = \text{constant}} \rightarrow ③$

The above result is not correct, since every charged particle when accelerated irradiates energy in the form of e-m waves . This effect is, of course, usually small and can be neglected.

• Solⁿ of eqn of motion (1) -

(a) Only in presence of uniform and static \vec{E} field -

$$① \Rightarrow m \frac{d\vec{v}}{dt} = q\vec{E} \Rightarrow \vec{v} = \frac{q}{m} \vec{E} dt$$

Integrating -

$$\left[\vec{v}(t) = \left(\frac{q}{m} \vec{E} \right) t + \vec{v}_0 \right] \quad \left(\vec{v}_0 = \vec{v}(t=0) \right) \quad (4a)$$

$$\Rightarrow \frac{d\vec{r}(t)}{dt} = \left(\frac{q}{m} \vec{E} \right) t + \vec{v}_0$$

$$\Rightarrow d\vec{r}(t) = \left(\frac{q}{m} \vec{E} \right) t dt + \vec{v}_0 dt.$$

Again integrating -

$$\vec{r}(t) = \frac{q}{m} \vec{E} \frac{t^2}{2} + \vec{v}_0 t + \vec{r}_0 \quad \left| \vec{r}_0 = \vec{r}(t=0) \right.$$

$$\text{or } \vec{r}(t) = \frac{1}{2} \left(\frac{q}{m} \vec{E} \right) t^2 + \vec{v}_0 t + \vec{r}_0 \quad \rightarrow (4b).$$

Where \vec{r}_0 and \vec{v}_0 denote the initial position vector and initial velocity at $t=0$ for the charged particle. The particle is moving with constant acceleration $\vec{a} = \frac{q\vec{E}}{m}$ in the dirⁿ of \vec{E} field, if $q > 0$ and in opposite to \vec{E} field, if $q < 0$.

There is no acceleration \perp^r to \vec{E} , so the particle's state of motion remains unchanged in a dirⁿ \perp^r to \vec{E} .

(3)

(B) In presence of only a uniform and static magnetic induction \vec{B} field.

$$\textcircled{1} \Rightarrow m \frac{d\vec{v}}{dt} = q \vec{v} \times \vec{B}$$

It is convenient to separate \vec{v} in components parallel ($\vec{v}_{||}$) and perpendicular (\vec{v}_{\perp}) to the magnetic field -

$$\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}$$

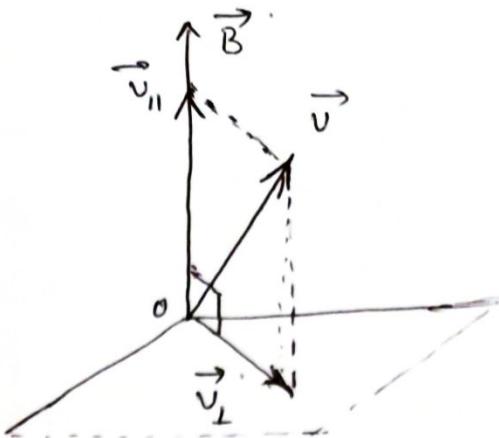


Fig-1

$$\text{Hence, } m \frac{d(\vec{v}_{||} + \vec{v}_{\perp})}{dt} = q (\vec{v}_{||} + \vec{v}_{\perp}) \times \vec{B}$$

$$\Rightarrow m \frac{d\vec{v}_{||}}{dt} + m \frac{d\vec{v}_{\perp}}{dt} = q (\underbrace{\vec{v}_{||} \times \vec{B}}_{=0} + q (\vec{v}_{\perp} \times \vec{B})).$$

$$\Rightarrow m \frac{d\vec{v}_{||}}{dt} + m \frac{d\vec{v}_{\perp}}{dt} = q (\vec{v}_{\perp} \times \vec{B}) \rightarrow \textcircled{5}.$$

$\because (\vec{v}_{\perp} \times \vec{B})$ is \perp^r upon the plane containing \vec{v}_{\perp} and \vec{B} , so it is \perp^r to \vec{B} , hence we can write the above eqnⁿ componentwise -

$$m \frac{d\vec{v}_{||}}{dt} = 0 \rightarrow \boxed{\frac{d\vec{v}_{||}}{dt} = 0} \xrightarrow{\textcircled{6a}} \vec{a}_{||} = 0 \quad (\text{parallel component})$$

$$\text{and } \boxed{m \frac{d\vec{v}_{\perp}}{dt} = q (\vec{v}_{\perp} \times \vec{B})} \rightarrow \textcircled{6b} \quad (\text{perpendicular component})$$

Eqnⁿ (6a) shows that the charged particle's initial velocity component along \vec{B} doesn't change with time and it is equal to the initial velocity component, i.e.

$$v_{||}(t=t_0) = v_{||}(t=0).$$

What does the eqnⁿ (6b) mean?

$$(6b) \rightarrow \vec{a}_\perp = \frac{d\vec{v}_\perp}{dt} = \vec{v}_\perp \times \left(\frac{q\vec{B}}{m} \right) \rightarrow (6c)$$

$$\vec{a}_\perp = \left(v_\perp \frac{qB}{m} \right) \hat{n}$$

= const.

From conservation of K.E. and constant $v_{||}$, the magnitude of acceleration \vec{a}_\perp is constant.

\hat{n} or the acceleration vector is \perp^r to both \vec{v}_\perp and \vec{B} (by 6c). This acceleration corresponds to ω_c (gyro-motion) of the particle with constant speed v_\perp on the plane perpendicular to \vec{B} .

Integrating (6c) \rightarrow

$$\vec{v}_\perp = \left(-\frac{qB}{m} \right) \times \vec{r}_c \rightarrow \boxed{\vec{v}_\perp = +\omega_c \times \vec{r}_c} \rightarrow (7)$$

$\omega_c \rightarrow$ called cyclotron frequency

or Larmor frequency or the gyro-frequency

$$\boxed{\omega_c = \frac{qB}{m}}$$

and \vec{r}_c is interpreted as the particle's position vector w.r.t. to a point C (the centre of gyration) on the plane \perp^r to \vec{B} . Since

ω_c and v_\perp are constants, $r_c = |\vec{r}_c|$ is also a constant.

The physical meaning is that the charged particle (2) is rotating about the 'instantaneous' centre C with the radius r_c and Larmor frequency ω_c in the plane \perp^r to \vec{B} . The 'instantaneous' centre of

$$v^2 = v_{||}^2 + v_\perp^2$$

$\frac{1}{2}mv^2 = KE \rightarrow$ conserved.

(by eqn (3))

$v_{||} = \text{const}$ by eqn (6a).

So, $v_\perp = |\vec{v}_\perp| \rightarrow \text{const}$.

(5)

gyration is called gyring centre and r_c is called.
Larmor radius / cyclotron radius / gyroradius.

$$\boxed{\omega_c = \frac{qB}{m}} \rightarrow \text{Larmor frequency} \rightarrow \textcircled{8}$$

$$\boxed{r_c = \frac{v_\perp}{\omega_c} = \frac{mv_\perp}{qB}} \rightarrow \text{Larmor radius} \rightarrow \textcircled{9}$$

$$\therefore \boxed{r_c = \frac{v_\perp}{\omega_c}} \rightarrow \textcircled{10}$$

As $|\vec{B}|$ incres $\rightarrow \omega_c$ incres.
 r_c decres.

$$Br_c = \frac{mv_\perp}{qB} = \frac{p_\perp}{qB}$$

Momentum per unit charge, called magnetic rigidity

Again, smaller the mass, the larger will be its gyrofrequency and the smaller its radius.

* r_c — for e^- and p^+ (for hydrogen plasma)
and constant v_\perp .

$$\frac{(r_c)_e}{(r_c)_p} = \frac{m_e v_\perp}{e B} \times \frac{e B}{m_p v_\perp} = \frac{m_e}{m_p} \left(\approx \frac{1}{1800} \right) = \frac{1}{1837}$$

i.e. The ratio of the two gyroradii for bN the two species is equal to the ratio of their masses.

$$[(r_c)_p > (r_c)_e]$$

$$(7) \Rightarrow \boxed{\omega_c = \frac{qB}{m}} - \textcircled{11}$$

(Using the right hand thumb rule or screw rule, the dir^n of

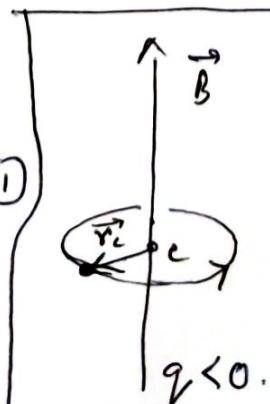
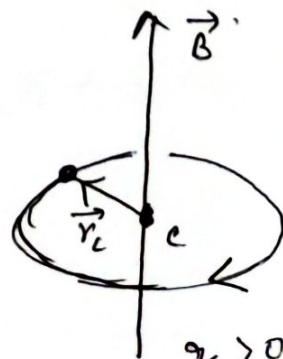


Fig-2 (Not in scale).



(6)

rotation (gyration) about \vec{B} can be visualized. For electrons $\vec{\omega}_c$ is parallel to \vec{B} and for ions/protons ω_c is antiparallel to \vec{B} .

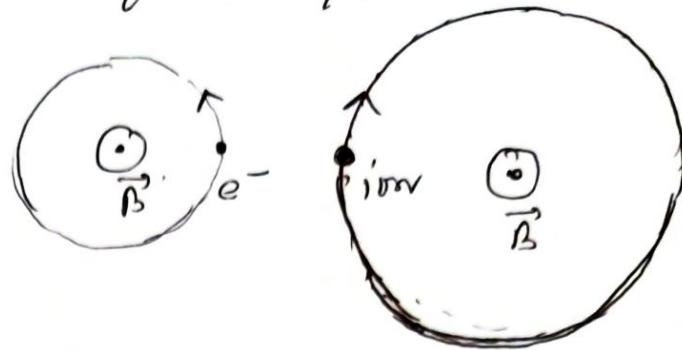


Fig - 3 (Magnetic field is facing out).

(2) Find gyrofrequency for e^- and p .

$$\left. \begin{array}{l} m_e = 9.1 \times 10^{-31} \text{ kg} \\ q_e = 1.6 \times 10^{-19} \text{ Coul.} \end{array} \right\} \rightarrow \omega_c = \frac{q_e B}{m_e} = 1.76 \times 10^{11} B \text{ (rad/s)}$$

$$\left. \begin{array}{l} m_p = 1.67 \times 10^{-27} \text{ kg} \\ q_p = 1.6 \times 10^{-19} \text{ C} \end{array} \right\} \rightarrow \omega_c = \frac{q_p B}{m_p} = 9.58 \times 10^7 B \text{ (rad/s)}$$

Since, $\vec{v}_{||}$ is constant and v_{\perp} is fixed along the circular path of radius r_c , so, the motion constitutes a helix along \vec{B} .

Ques. Dirn of $\vec{v}_{||}$ is along \vec{B} independent of the charge of the particles, so, the guiding centre will slide along \vec{B} (like a bead sliding along a st. wire) with speed $v_{||}$.

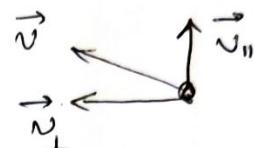
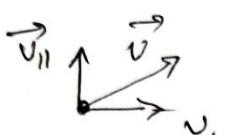
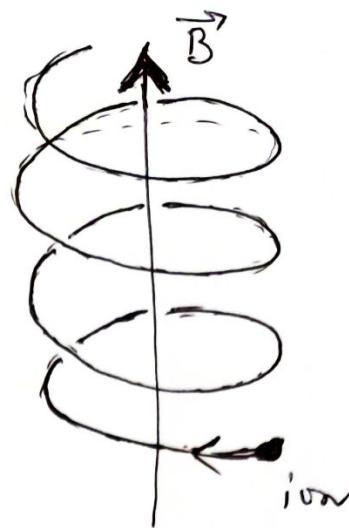
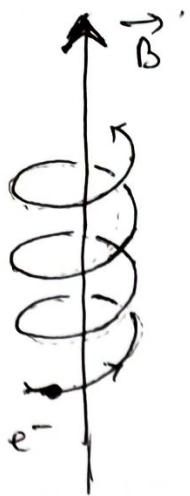


Fig-4 - Helicoidal trajectory of e^- and ion particles in a uniform magnetic field.

* If you point your thumb along the dirⁿ of the magnetic field, the finger of your left hand curl in the dirⁿ of rotation of positively charged ions, while those of your right hand to do the same for e^- s.

• The motions of the charged particles are equivalent to currents and the dirⁿ of

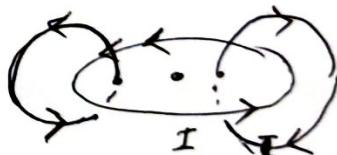


Fig-5

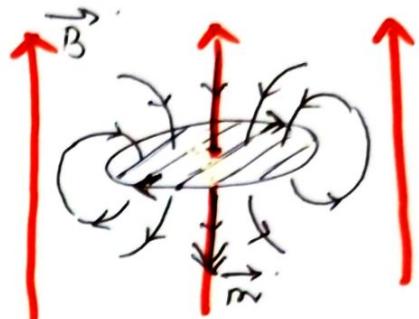
These currents is are such that magnetic field produced by these tiny current reduce the ambient magnetic field inside the particle orbits. High-pressure plasmas reduce the externally imposed magnetic field through the superposition of this 'diamagnetic' effect from a high density of energetic particles.

(8)

- If $\vec{B} = 0$, $v_{\parallel} = 0$, i.e. the particle (charged) is thrown \perp^r to the ^{uniform} magnetic field, the particle is rotating with constant speed v_{\perp} ($= v$) on the plane \perp^r to the magnetic field with angular frequency $\omega_c = \frac{qB}{m}$ and in a circular orbit of radius $r_c = \frac{mv}{qB}$.

- Magnetic Moment (\vec{m}):-

Magnitude $|\vec{m}| = \text{current} \times \text{orbital area}$
 $\Rightarrow I A$



The magnetic moment \vec{m} associated with the current (circulating) I is fig-6 normal to the area (A) bounded by the particle orbit and points in the dir^n opposite to the externally applied \vec{B} field as shown in fig-6.

This circulating current is $I = \frac{|q|}{T_c} = \frac{|q| \omega_c}{2\pi}$,

where $T_c = \frac{2\pi}{\omega_c}$ is the time period of rotation in the orbit, known as cyclotron period or Larmor period.

$$\therefore |\vec{m}| = \frac{|q| \omega_c}{2\pi} \times \pi r_c^2 = \frac{1}{2} |q| \omega_c r_c^2.$$

Using the relations for ω_c and r_c from (8) and (9) -

$$|\vec{m}| = \frac{1}{2} |q| \left(\frac{|q| B}{m} \right) \left(\frac{mv_{\perp}}{|q| B} \right)^2$$

$$= \frac{\frac{1}{2} mv_{\perp}^2}{B}$$

$$KE = \frac{1}{2} mv^2 = \frac{1}{2} mv_{\parallel}^2 + \frac{1}{2} mv_{\perp}^2$$

or $|\vec{m}| = \frac{m \omega_{\perp}}{B}$, where ω_{\perp} is the part of KE associated with the transverse velocity v_{\perp} .

(9)

Thus in vector form -

$$\vec{m} = \frac{\omega_{\perp}}{B} (-\hat{B}), \quad \hat{B} \rightarrow \text{unit vector along } \vec{B},$$

$$\therefore \boxed{\vec{m} = -\frac{\omega_{\perp}}{B^2} \vec{B}} \rightarrow (12).$$

Magnetization (\vec{M}):

\rightarrow Magnetic moment per unit volume

If n are n number of magnetic dipoles of magnetic moment \vec{m} in unit volume, then

$$\boxed{\vec{M} = n \vec{m} = -\left(\frac{n \omega_{\perp}}{B^2}\right) \vec{B}} \rightarrow (13)$$

$n \omega_{\perp} \rightarrow k_B$ per unit volume associated with the transverse particle velocity.

- Again $\boxed{\vec{M} = \chi_m \vec{H}}$ $\rightarrow (14)$,

where χ_m is called the magnetic susceptibility of the plasma medium.

Since by eqn (13), $|\vec{M}| \propto \frac{1}{|B|}$ and by eqn (14),

$|\vec{M}| \propto |\vec{H}|$, so, the relation between H and B is not linear. Within this context it is generally not convenient to treat a plasma as a magnetic medium.

(10)

(c) In presence of both \vec{E} and \vec{B} fields, which are static and uniform.

We consider now the motion of a charged particle in presence of both electric and magnetic fields that are constant in time and spatially uniform. The non-relativistic eqnⁿ of motion:-

$$m \frac{d\vec{v}}{dt} = q [\vec{E} + \vec{v} \times \vec{B}] \quad \text{--- (1)}$$

Taking components parallel & perpendicular to \vec{B} ,

$$\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}, \quad \vec{E} = \vec{E}_{||} + \vec{E}_{\perp},$$

We can resolve the above eqnⁿ (1) into two components equations:-

$$m \frac{d\vec{v}_{||}}{dt} = q \vec{E}_{||} \quad \rightarrow (15)$$

$$\text{and } m \frac{d\vec{v}_{\perp}}{dt} = q (\vec{E}_{\perp} + \vec{v}_{\perp} \times \vec{B}). \quad \left[\because \vec{v}_{||} \times \vec{B} = \vec{0} \right] \quad \rightarrow (16).$$

Applying the similar treatment as in (a),

$$\begin{aligned} (4a) \rightarrow \boxed{\vec{v}_{||}(t) = \left(\frac{q \vec{E}_{||}}{m} \right) t + \vec{v}_{0||}} &\rightarrow (17a) \quad \vec{v}_{||}(t=0) = \vec{v}_{0||} \\ (4b) \rightarrow \boxed{\vec{r}_{||}(t) = \frac{1}{2} \left(\frac{q \vec{E}_{||}}{m} \right) t^2 + \vec{r}_{0||} t + \vec{r}_{0||}} &\rightarrow (17b) \quad \vec{r}_{||}(t=0) = \vec{r}_{0||} \\ &\quad \vec{v}(t) = \vec{v}_{||}(t) + \vec{v}_{\perp}(t) \\ &\quad \vec{r}(t) = \vec{r}_{||}(t) + \vec{r}_{\perp}(t) \end{aligned}$$

Now we make a velocity transformation in eqnⁿ (16) corresponding to \vec{v}_{\perp} in such a manner that the electric field is transformed away, whereas the magnetic

(11)

field is left unchanged.

$$\vec{v}_\perp \longrightarrow v'_\perp(t) = \vec{v}_\perp(t) - \vec{v}_{EM}$$

Then $\vec{v}_{EM} \parallel \vec{v}_\perp(t)$ and also a uniform velocity vector \vec{v}'_\perp represents the particle's velocity w.r.t. a inertial frame moving with uniform velocity \vec{v}_{EM} .

(16) $\rightarrow m \frac{d\vec{v}_\perp}{dt} + m \underbrace{\frac{d\vec{v}_{EM}}{dt}}_{=0} = q \left[\vec{E}_\perp + (\vec{v}'_\perp + \vec{v}_{EM}) \times \vec{B} \right]$

$$\Rightarrow m \frac{d\vec{v}'_\perp}{dt} = q (\vec{v}'_\perp \times \vec{B}) + q \left[\vec{E}_\perp + (\vec{v}_{EM} \times \vec{B}) \right]$$

To remove \vec{E}_\perp , we put

$$\vec{E}_\perp = \vec{B} \times \vec{v}_{EM} \rightarrow \vec{E}_\perp \times \vec{B} = (\vec{B} \times \vec{v}_{EM}) \times \vec{B}$$

$$\boxed{\begin{aligned} & (\vec{A} \times \vec{B}) \times \vec{C} \\ & = (\vec{A} \cdot \vec{C}) \vec{B} - (\vec{B} \cdot \vec{C}) \vec{A} \end{aligned}} \quad \Rightarrow \vec{v}_{EM} = \frac{\vec{E}_\perp \times \vec{B}}{B^2} \quad \boxed{\begin{aligned} & \vec{v}_{EM} \parallel \vec{v}_\perp \\ & \text{and } \vec{v}_\perp \perp \vec{B} \\ & \text{so, } \vec{v}_{EM} \perp \vec{B} \end{aligned}}$$

The physical meaning is that if the reference frame is moving with uniform velocity \vec{v}_{EM} as given by $\boxed{\vec{v}_{EM} = \frac{\vec{E}_\perp \times \vec{B}}{B^2}}$, $\rightarrow (18)$

then an observer in that frame observes the motion of the particle in the plane normal to \vec{B} which is entirely governed by. —

$$\boxed{m \frac{d\vec{v}'_\perp}{dt} = q (\vec{v}'_\perp \times \vec{B})} \rightarrow (19)$$

which is identical to (6b) and implies that in the

(12)

reference frame moving with uniform velocity \vec{v}_{EM} , given by (18), the charged particle describes a circular motion at the cyclotron frequency, ω_c with radius r_c ,

$$\vec{v}_\perp = \vec{\omega}_c \times \vec{r}_c \quad (\text{by (7)}) \rightarrow (20).$$

thus, the particle velocity can be expressed in vector form

$$\vec{v} = \vec{v}_{||} + \vec{v}_\perp$$

$$= \vec{v}_{||} + \vec{v}_\perp' + \vec{v}_E$$

$$= \left[\left(\frac{q \vec{E}_{||}}{m} \right) t + \vec{v}_{0||} \right] + \underbrace{(\vec{\omega}_c \times \vec{r}_c)}_{\text{by (20)}} + \underbrace{\frac{\vec{E}_\perp \times \vec{B}}{B^2}}_{\text{by (18)}}$$

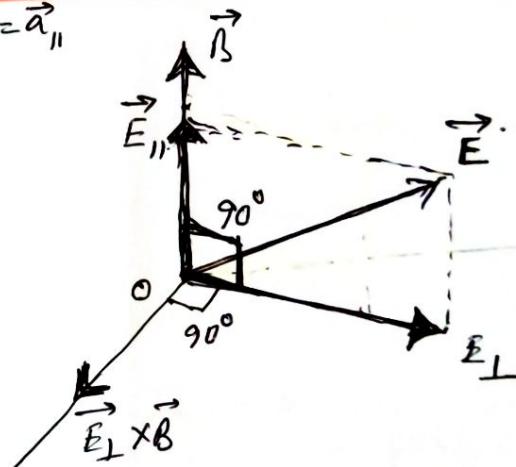
$$\therefore \vec{v} = \vec{\omega}_c \times \vec{r}_c + \frac{\vec{E}_\perp \times \vec{B}}{B^2} + \left(\frac{q \vec{E}_{||}}{m} \right) t + \vec{v}_{0||} \rightarrow (21).$$

Let us view the velocity term \vec{v}_{EM} in the context of the motion of a charge particle. \vec{v}_{EM} is independent of mass and charge of the particles.

$$\text{Hence, } \vec{E}_{||} \times \vec{B} = 0, \vec{v}_{EM}$$

can also be expressed as -

$$\boxed{\vec{v}_{EM} = \frac{\vec{E} \times \vec{B}}{B^2}}, \quad \left(\because \vec{E} = \vec{E}_{||} + \vec{E}_\perp \text{ & by eqn (18)} \right) \rightarrow (22).$$



(fig- 7).

Due to this velocity term \vec{v}_{EM} present in eqn (21), the guiding centre of the gyro-motion of the

(13)

charge particle about \vec{B} as given by the 1st term $\vec{\omega}_c \times \vec{r}_c$ in RHS of (21), will be drifted along $\vec{E} \times \vec{B}$ dir \sim (i.e. \perp^n to \vec{E} and \vec{B}) with constant speed $\left| \frac{\vec{E} \times \vec{B}}{B^2} \right|$, so the resulting motion of the charge particle is a cycloid ~~confined on~~
line \perp^n to \vec{E} and \vec{B} . So, \vec{v}_{EM} is usually called the plasma drift velocity or e-m plasma drift or $\vec{E} \times \vec{B}$ drift.

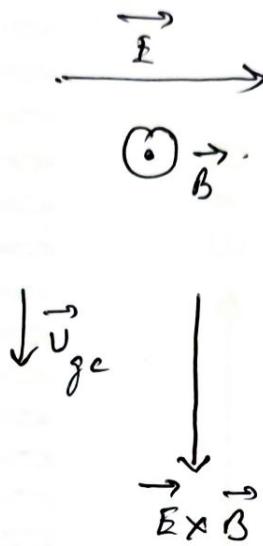
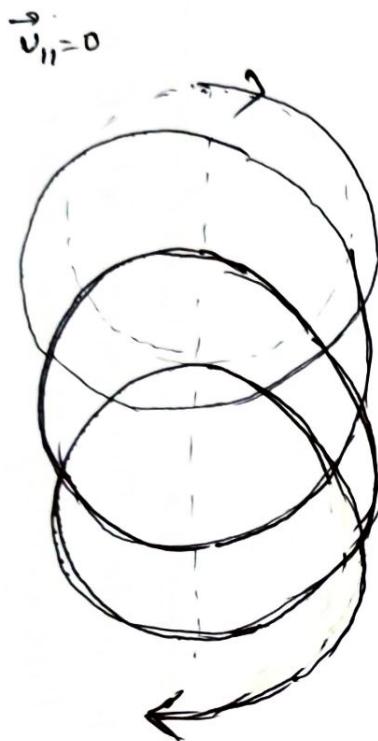
The third term in RHS of (21) indicates that the guiding centre of the gyration is accelerating with constant acceleration along \vec{B} for +ve charged ion and opposite to \vec{B} for -ve charged e^- ($\because \vec{E}_\parallel \parallel \vec{B}$), and the fourth term is the initial velocity of the particle along \vec{B} .

To get a more physical picture about the cycloidal trajectories, we choose a +ve ion motion! The +ve ion is accelerated in the direction of \vec{E} during one part of its gyro orbit, but it gets decelerated during the other part, as its $d\vec{r} \sim$ gets reversed. The result of these accelerations and decelerations is that the radii of curvature of the gyro-orbits will be slightly longer on

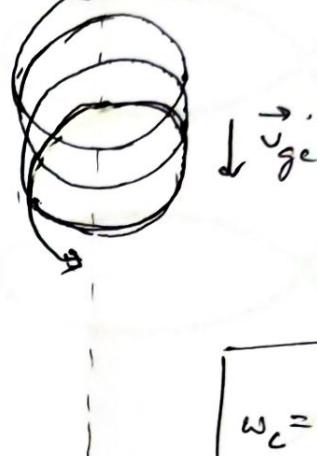
$$\begin{aligned} & \vec{E} \perp \vec{B} \\ & \therefore \vec{E}_\parallel = 0 \\ & \vec{a} = \frac{q \vec{E}_\parallel}{m} \\ & = 0 \end{aligned}$$

(21)

The side where the particles have greater KE than on the side where the particles have less KE. This gives rise to the drift l^r to E as illustrated in the figure.



$$\gamma_c = \frac{m v_{\perp}}{q B} \quad \text{by eqn (9)}$$



$$w_c = \frac{q B}{m} \quad \text{by eqn (8)}$$

$$\vec{v}_{\parallel} = 0 \rightarrow \vec{v} = w_c \times \vec{r}_c + \frac{\vec{E} \times \vec{B}}{B^2} \quad (21)$$

Eqn 8-($E \times B$) drift for ion and e^- . The half orbit for the left-hand side is larger than that on the right-hand side for the e^- motion, and it is simply reversed for the ion motion.

It is observed in the expression of \vec{v}_{EM} in (22) that \vec{v}_{EM} is independent of q , m , \vec{v}_{\parallel} and \vec{v}_{\perp} . This means that the whole plasma drifts together across the electric and magnetic fields with the same velocity (as shown in the above figure), \vec{v}_{EM} (the accelerating term $\frac{q E_{\parallel}}{m}$ in eqn (21) is zero). The e^- 's w_c is small compared to ion, and hence drift less per cycle. But their w_c is large, and the two effects exactly cancel. So, e^- and ion suffer same drift in same direction.

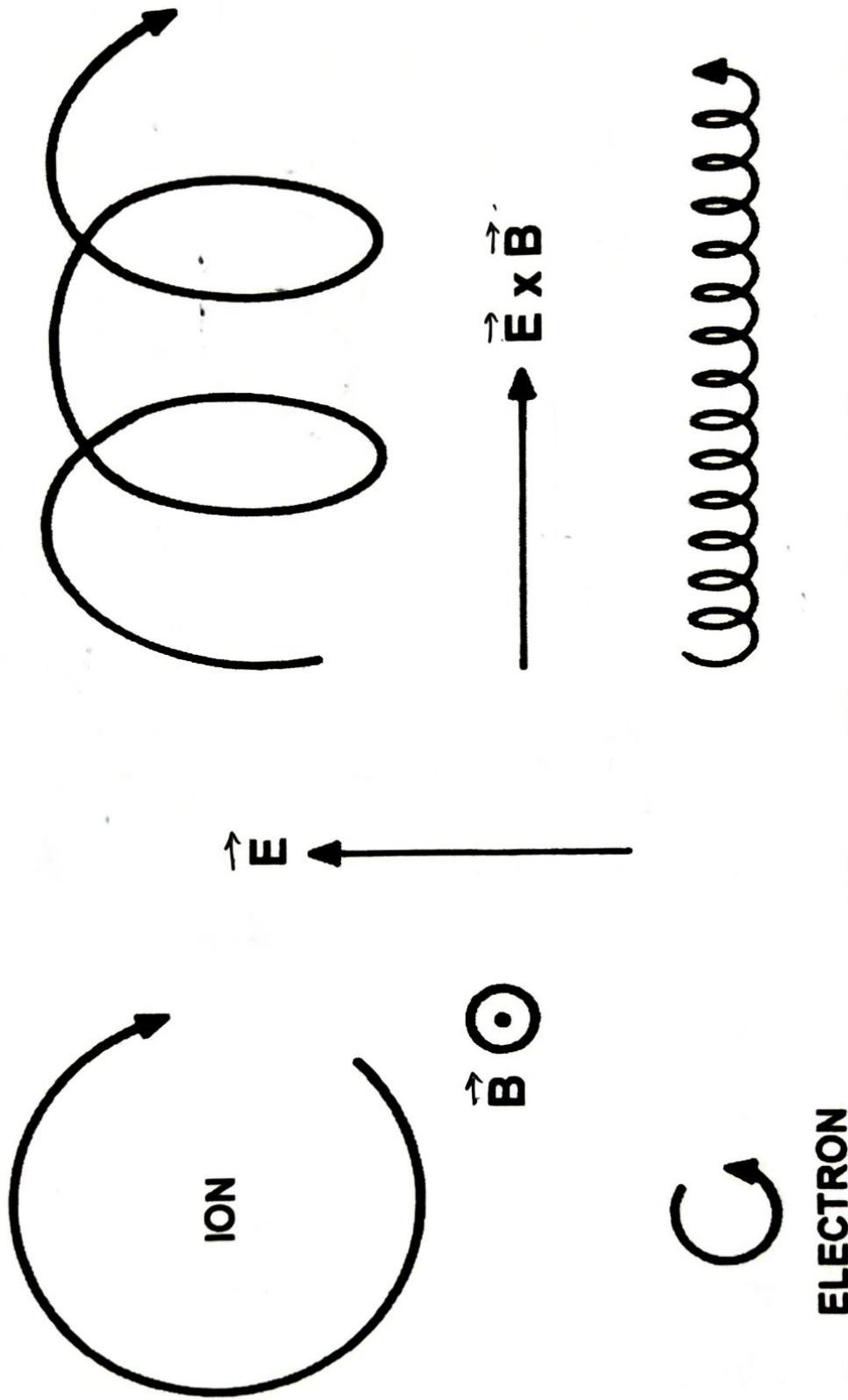
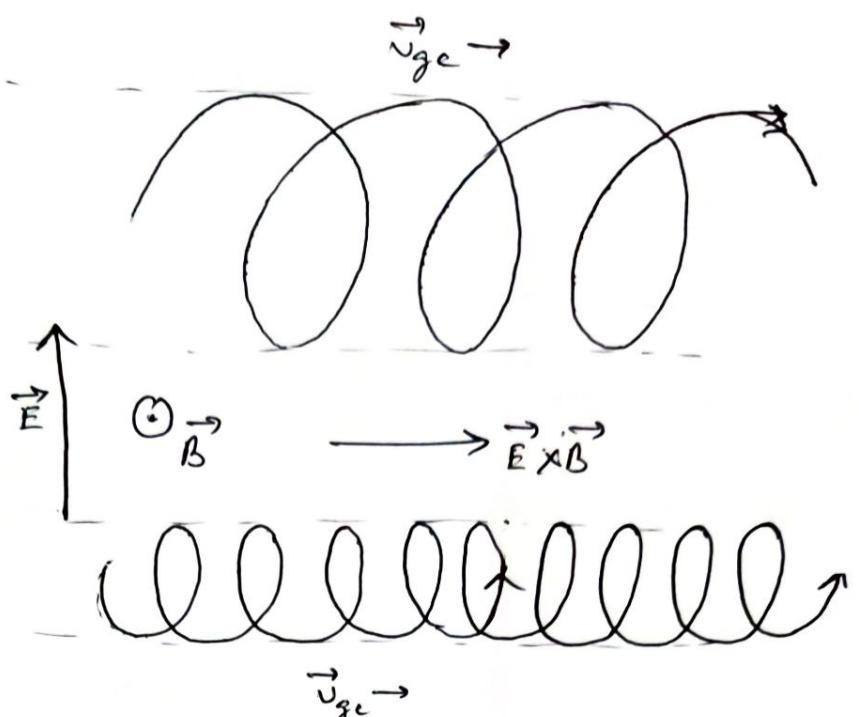
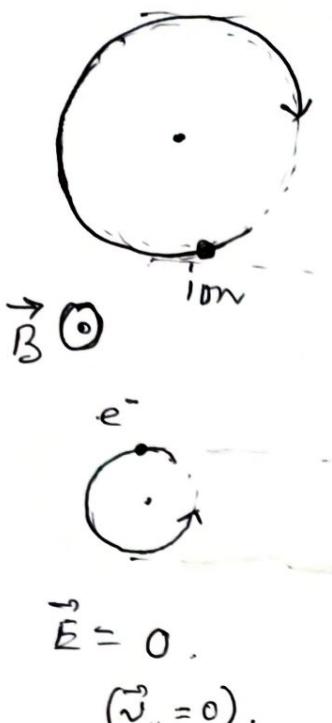


Fig. Cycloidal trajectories described by ions and electrons in crossed electric and magnetic fields. The electric field \vec{E} acting together with the magnetic flux density \vec{B} gives rise to a drift velocity in the direction given by $\vec{E} \times \vec{B}$.



$$\vec{E} = 0.$$

$$(\vec{v}_{||} = 0).$$

$$(\vec{v}_n = 0) \quad (2) \Rightarrow \vec{v} = \vec{\omega}_c \times \vec{r}_c + \frac{\vec{E} \times \vec{B}}{B^2}.$$

Fig-9. (same as fig 8). Cycloidal trajectories described by ion and e⁻s in crossed electric and magnetic fields. \vec{E} with \vec{B} gives rise to the drift velocity \vec{v}_{EM} in the dirⁿ given by $(\vec{E} \times \vec{B})$.

In Summary: The motion of a charge particle in stationary and uniform \vec{E} and \vec{B} fields consists of three components : - [See eqn (21)]

①. $\vec{v}_{||t} = \left(\frac{q \vec{E}_{||}}{m} \right) t + \vec{v}_{0||} \rightarrow$ the particle moves with constant acceleration $\frac{q \vec{E}_{||}}{m}$ along the \vec{B} field. If $\vec{E}_{||} = 0$ (i.e. if $\vec{E} \perp \vec{B}$), the particle moves with uniform velocity $\vec{v}_{0||}$ (initial velocity) along \vec{B} .

② $\vec{v}_c = \vec{\omega}_c \times \vec{r}_c \rightarrow$ a gyration/cyclotron motion on the plane $\perp r$ to \vec{B} at the cyclotron frequency $\omega_c = \frac{qB}{m}$ and radius $r_c = \frac{v_0}{\omega_c}$.

$$③ \vec{v}_{EM} = \frac{\vec{B} \times \vec{B}}{B^2} \rightarrow a \vec{E} \times \vec{B} \text{ drift velocity,}$$

perpendicular to both \vec{B} and \vec{E} .

The overall picture is: - the 3D orbit (trajectory) for a charged particle is a slanted helix towards $\vec{E} \times \vec{B}$ with changing pitch.

④ In presence of an external force \vec{F} along with \vec{E} and \vec{B} fields (stationary and uniform):

In case of a non-inertial system, pseudo force \vec{F} will act on the charged particles in plasma along with the Lorentz force (e.g. in presence of gravitational force). So, the eqn of motion for a charged particle becomes -

$$m \frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) + \vec{F} \rightarrow ②③.$$

(non-relativistic case)

$$\Rightarrow m \frac{d\vec{v}}{dt} = q\vec{E} + q\left(\frac{\vec{F}}{q}\right) + q(\vec{v} \times \vec{B}) \rightarrow ②③'$$

Thus, the effect of this force \vec{F} , is analogous to the effect of \vec{E} field on the charge particle. If we assume here that \vec{F} is uniform and constant (stationary), the drift produced by the force \vec{F} is given by -

$$\vec{v}_F = \frac{\vec{F}_\perp \times \vec{B}}{qB^2}, \frac{\vec{F} \times \vec{B}}{qB^2} \rightarrow ②④. / \quad \vec{F} = \vec{F}_\perp + \vec{F}_\parallel$$

$$\vec{F}_\parallel \parallel \vec{B}$$

$$\vec{F}_\perp \perp \vec{B}$$

(166)

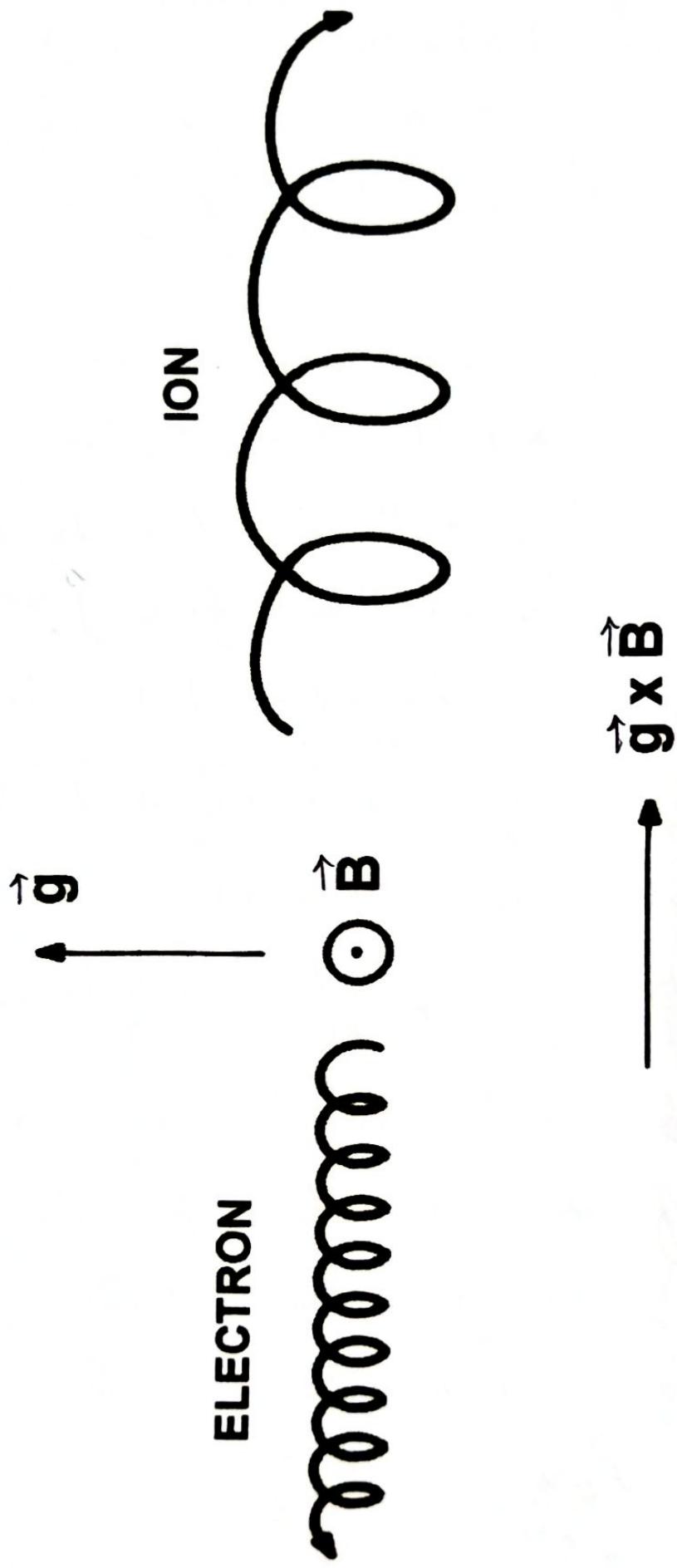


Fig. Drift of a gyrating particle in crossed gravitational and magnetic fields.

Example - In case of a uniform gravitational field -

$\vec{F} = m\vec{g}$, when $\vec{g} \rightarrow \text{act.}$ due to gravity.

$$\therefore \text{Drift velocity } \vec{v}_g = \frac{m}{q} \frac{\vec{g} \times \vec{B}}{B^2} \rightarrow (25)$$

This drift velocity depends on the ratio $\frac{m}{q}$ and therefore it is in opposite directions for particles of opposite charge. (Fig-10).

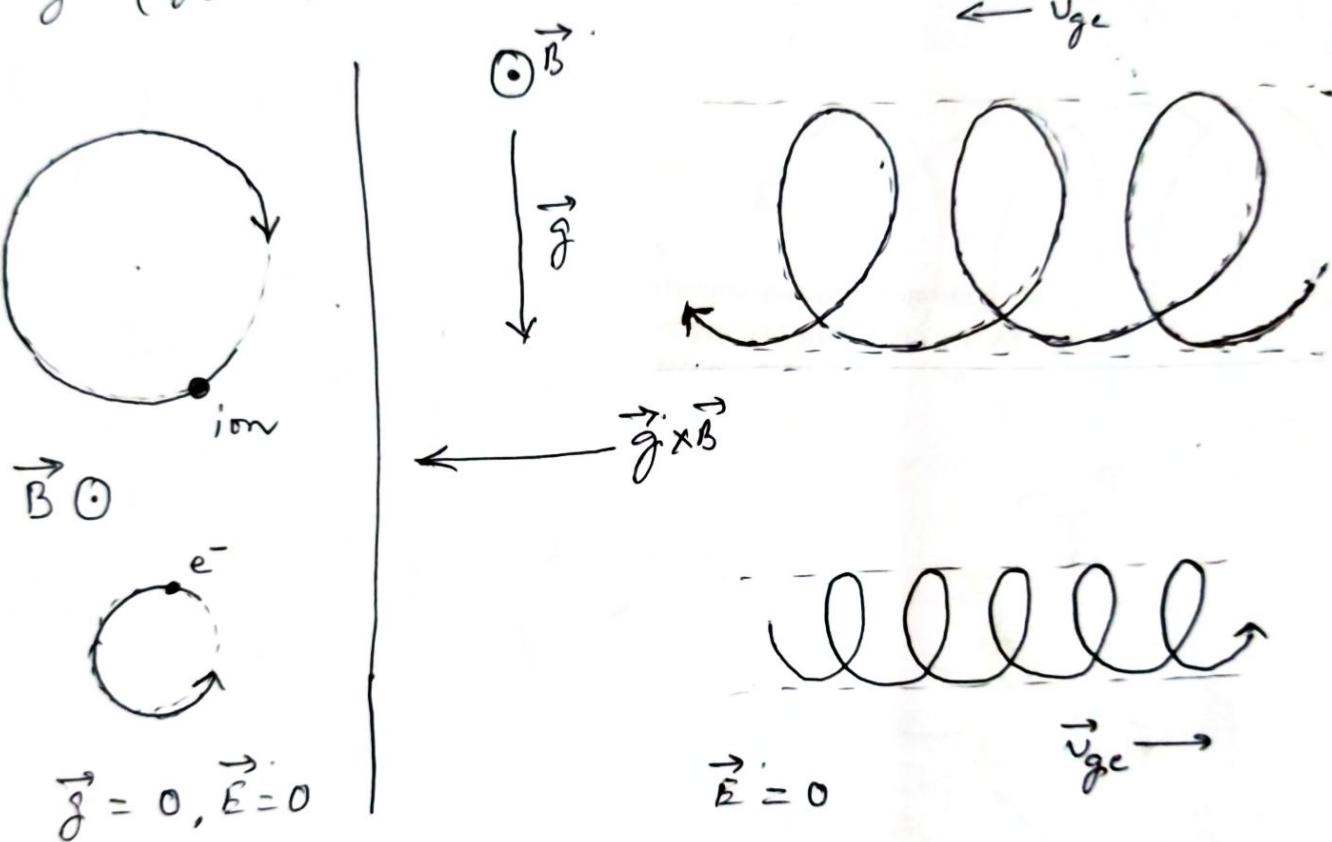
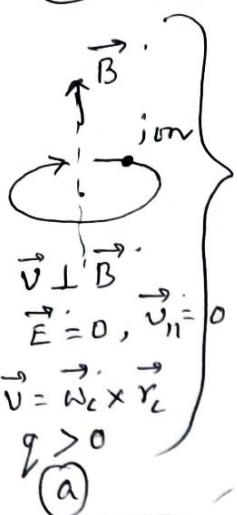


Fig-10. Drift of an ion and an electron in crossed gravitational and magnetic fields along $\vec{g} \times \vec{B}$ dirⁿ.

By eqn (25), \vec{v}_g , unlike \vec{v}_{EM} , depends on charge and mass. So, the presence of gravity gives rise to a net current in a plasma; the ions ~~are~~ drift one ways and the

17b



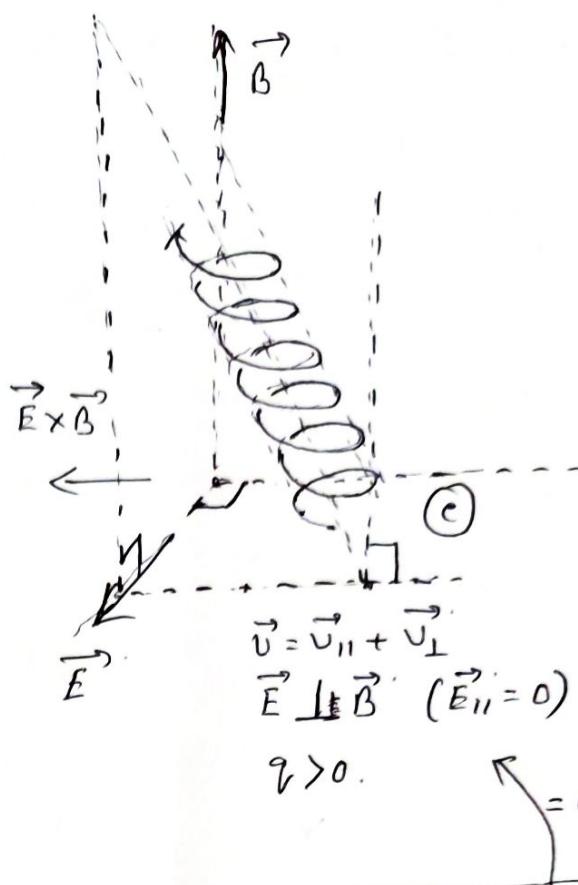
$$\boxed{\vec{v} = \vec{\omega}_c \times \vec{r}_c}$$



$$\boxed{\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}}$$

$$\vec{E} = 0$$

$$\boxed{\vec{v} = \vec{\omega}_c \times \vec{r}_c + \vec{v}_{0||}}$$

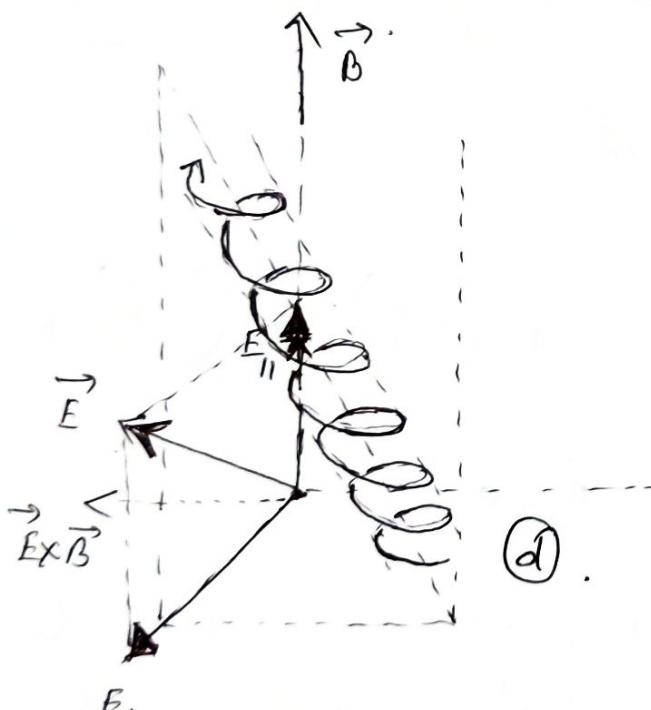


$$\boxed{\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}}$$

$$\boxed{\vec{E} \perp \vec{B} \quad (\vec{E}_{||} = 0)}$$

$$(2) \Rightarrow \boxed{\vec{v} = \vec{\omega}_c \times \vec{r}_c + \frac{\vec{E} \times \vec{B}}{B^2} + \left(\frac{q E_{||}}{m} \right) t + \vec{v}_{0||}}$$

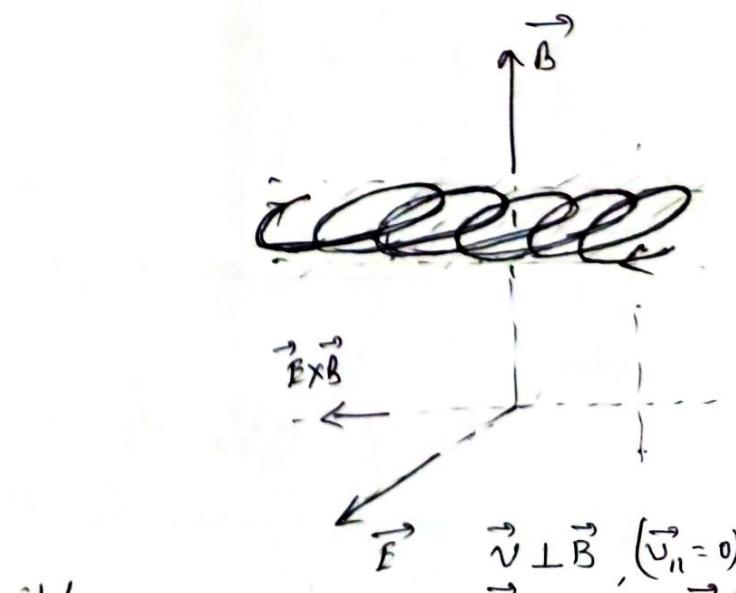
constant.
(at $t=0$)



$$\boxed{\vec{v} = \vec{v}_{||} + \vec{v}_{\perp}}$$

$$\boxed{\vec{E} = \vec{E}_{||} + \vec{E}_{\perp}}$$

$$q > 0.$$



$$\boxed{\vec{v} = \vec{\omega}_c \times \vec{r}_c + \frac{\vec{E} \times \vec{B}}{B^2}}$$

Fig - Slanted helix with changing pitch.

e^- s drift the other — The ions, which are much heavier, drift much faster. In a finite plasma, this current therefore gives rise to charge separation.

(?) The plasma 'cloud' above the Earth does not seem to fall down due to gravity. Why?

It is because the gravitational drift occurs on the charged particles along $\vec{g} \times \vec{B}$, so, the charged particles ~~fall~~ are drifted horizontally. The gravitational drift separates e^- s and ions and so an electric field builds up in horizontal direction L^r to \vec{B} and the plasma ~~does not~~ does indeed drift downward, after all due to $\vec{E} \times \vec{B}$ drift.

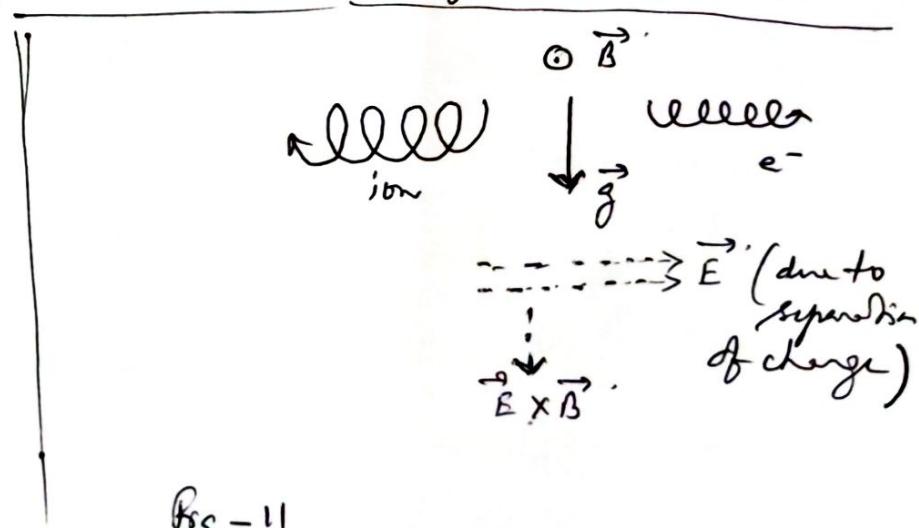


Fig - 11

(1)

PART-II

Motion of charged particles in non-uniform magnetic fields.

HJ

- When the fields are spatially nonuniform, or when they vary with time, the rigorous analytic expressions for the trajectories of the charged particles can not be obtained in a closed form. It becomes a challenge for the physicists to solve the eqns of motion for those particles, as those equations become non-linear. For such a situation, numerical methods have to be used to obtain all the details of the motions.

- But we can also adopt an approximate method, if the details of the particle motion are not of interest. An approximate method will work when the magnetic field is strong and slowly varying in both space and time, and when the electric field is weak. Slowly varying in space and time means that the fields are almost uniform and constant, at least on the distance and time scales seen by a charged particle during one gyration about the magnetic field.

(2)

- Of course, we will investigate the motion of charged particles in static magnetic field slightly inhomogeneous in space. The word slightly means here that the spatial variation of magnetic induction \vec{B} field inside the gyro-orbit of a charged particle about the magnetic field is very much small compared to the magnitude of \vec{B} . In a distance r_c , the

spatial change
in magnetic
field -

$$|\delta B| = r_c |\vec{\nabla} B|,$$

when $\vec{\nabla} B$ is the

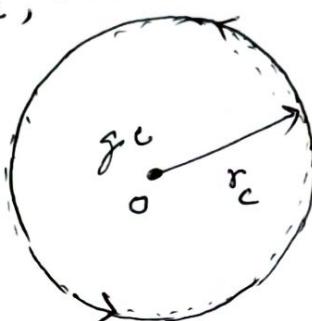
gradient of magnitude B_{sg}

of \vec{B} , r_c (or r_L) - is the gyro-radius/Larmour radius.

For our approximation method, we assume that $\delta B = r_c |\vec{\nabla} B| \ll |\vec{B}|$

$$\text{or } r_c \frac{|\vec{\nabla} B|}{B} \ll 1$$

- If the deviations from uniformity are small ($\delta B \ll B$), we can solve the eqn of motion



In static & uniform
 \vec{B} field



In static and
inhomogeneous
(slightly) \vec{B} field.

for the particle trajectory only in the 1st-order approximation. The analysis of charged particle motion in stationary fields based on this approximation is often referred to as the first-order orbit theory.

Some time Debye was first used systematically by the Swedish Scientist Alfvén, and it is also known as Alfvén Approximation or the guiding centre approximation.

- In this particular theory, the concept of guiding centre is of great utility. For .. a static and uniform magnetic field, the motion of a charge particle can be regarded as .. a superposition of gyromotion about the magnetic lines of force with a uniform motion of the guiding centre along the magnetic field lines ($\vec{v} = \vec{v}_\perp + \vec{v}_{||} = \vec{\omega}_c \times \vec{r}_c + \vec{v}_{||}$). If we suppose that the particle is in motion on a plane \perp to the field line along a circular path about the field lines, an introduction of slight ^{spatial} inhomogeneity to the magnetic field will change the complete circular motion to a nearly circular motion.

(fig-1)

(4)

Thus, due to the spatial variation of \vec{B} , there is a gradual drift of the guiding centre across \vec{B} as well as a gradual change of its velocity along \vec{B} .

- The rapid gyrations of the charged particles about \vec{B} are not usually of great importance. So, those motions are eliminated from the equation of motion and motions of the guiding centres are more important. To study the ionised gas permeated by an inhomogeneous magnetic field. The small oscillations in the motion of guiding centre during one gyration period is averaged out, since they represent the effect of perturbations due to the spatial variation of the magnetic field. Thus the problem is reduced to the calculation of the average values over one gyration period (and not the instantaneous values) of the guiding centre motion. So, this type of theory is called guiding-centre approximation.

(5)

Equation of motion in the first-order approximation:

Let \vec{B}_0 be the magnetic flux density of the pervaded magnetic field at the guiding centre, which is again the origin of the guiding centre coordinate system at $t=0$.

The magnetic field near the origin (or ^{near the} guiding centre) can be expressed by a Taylor expansion about the origin, i.e.

$$\vec{B}(\vec{r}) = \vec{B}_0 + \vec{r} \cdot (\vec{\nabla} \vec{B}) + \dots \rightarrow (2)$$

Where the derivatives of \vec{B} are to be calculated at the origin. (It should be remembered that the instantaneous position of the particle's guiding centre actually moves slightly during one period of rotation, while the origin is kept fixed during this time.)

Since, we are assuming that the spatial variation of \vec{B} in a distance of the order of Larmor radius is much smaller than the magnitude of \vec{B} itself, the higher order terms in (2) can be neglected.

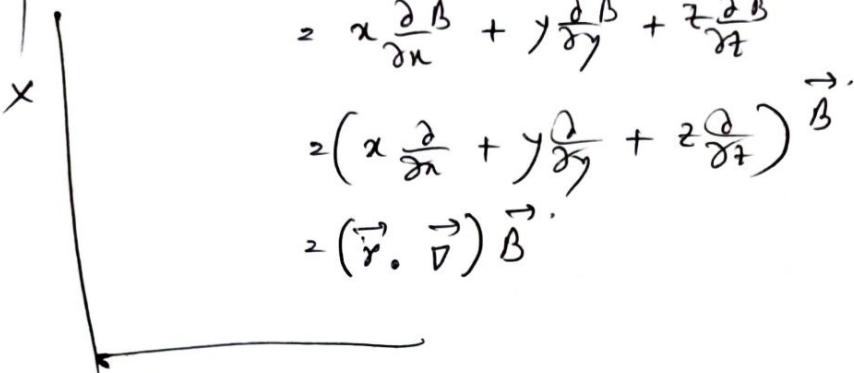
Thus, the magnetic field at the particle's position (\vec{r}) differs only slightly from that existing at the guiding centre., i.e.

$$\therefore \vec{B}(\vec{r}) = \vec{B}_0 + \vec{r} \cdot (\vec{\nabla} \vec{B}). \rightarrow (3).$$

$$(\because |\vec{r} \cdot (\vec{\nabla} \vec{B})| = \delta B \ll B)$$

$\vec{r} \cdot (\vec{\nabla} \vec{B})$ can be written explicitly as $(\vec{r} \cdot \vec{\nabla}) \vec{B}$: (6)

$$\begin{aligned}\vec{r} \cdot (\vec{\nabla} \vec{B}) &= (\hat{i}x + \hat{j}y + \hat{k}z) \cdot \left(\hat{i} \frac{\partial \vec{B}}{\partial x} + \hat{j} \frac{\partial \vec{B}}{\partial y} + \hat{k} \frac{\partial \vec{B}}{\partial z} \right) \\ &= x \frac{\partial \vec{B}}{\partial x} + y \frac{\partial \vec{B}}{\partial y} + z \frac{\partial \vec{B}}{\partial z} \\ &= \left(x \frac{\partial}{\partial x} + y \frac{\partial}{\partial y} + z \frac{\partial}{\partial z} \right) \vec{B} \\ &= (\vec{r} \cdot \vec{\nabla}) \vec{B}.\end{aligned}$$



Then the eqn of motion of an charged particle in the slightly inhomogeneous ^{static} magnetic field becomes -

$$\begin{aligned}m \frac{d\vec{v}(\vec{r})}{dt} &= q(\vec{v}_0 \times \vec{B}) \quad (\text{non-relativistic}) \\ &= q \left\{ \vec{v}(\vec{r}) \times \left[\vec{B}_0 + \vec{r} \cdot (\vec{\nabla} \vec{B}) \right] \right\} \\ &= q \left\{ \vec{v}(\vec{r}) \times \vec{B}_0 \right\} + q \left\{ \vec{v}(\vec{r}) \times [\vec{r} \cdot (\vec{\nabla} \vec{B})] \right\} \\ &\rightarrow (4)\end{aligned}$$

The particle velocity can be written as a superposition.

$$\vec{v}(\vec{r}) = \vec{v}_0(\vec{r}) + \vec{v}_1(\vec{r}) \rightarrow (5).$$

where \vec{v}_0 is the solution of zero-order eqn of motion,
i.e. sum of $m \frac{d\vec{v}_0}{dt} = q(\vec{v}_0 \times \vec{B}_0)$,

and \vec{v}_1 is a first-order perturbation,
 $|\vec{v}_1| \ll |\vec{v}_0|$.

Putting ⑤ in ④ and neglecting the second-order 7.

$$\vec{v} \times \vec{B}_0 = (\vec{v}_0 + \vec{v}_1) \times \vec{B}_0 = \vec{v}_0 \times \vec{B}_0 + \vec{v}_1 \times \vec{B}_0$$

$$\vec{v} \times [\vec{r} \cdot (\vec{\nabla} \vec{B})] = (\vec{v}_0 + \vec{v}_1) \times \left[(\vec{r}_0 + \vec{r}_1) \cdot \vec{\nabla} \vec{B} \right]$$

$$= \vec{v}_0 \times [\vec{r}_0 \cdot (\vec{\nabla} \vec{B})] + \vec{v}_1 \times [\vec{r}_1 \cdot (\vec{\nabla} \vec{B})]$$

$$+ \vec{v}_1 \times [\vec{r}_0 \cdot (\vec{\nabla} \vec{B})] - \vec{v}_0 \times [\vec{r}_1 \cdot (\vec{\nabla} \vec{B})]$$

Second-order terms.

Thus, we have got the eqn of motion of the charged particle under first-order approximation.

$$\cancel{m \frac{d\vec{v}}{dt} + m \frac{d\vec{v}_1}{dt} = q(\vec{v}_0 \times \vec{B}_0) + q(\vec{v}_1 \times \vec{B}_0) + q \vec{v}_0 \times [\vec{r}_0 \cdot (\vec{\nabla} \vec{B})]} .$$

$$\boxed{m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}_0) + q \vec{v}_0 \times [\vec{r}_0 \cdot \underline{\vec{\nabla} \vec{B}}]} \rightarrow ⑥$$

This eqn looks like the eqn of motion of a charged particle in an homogeneous and static magnetic field \vec{B}_0 under the influence of a ^{uniform} external force \vec{F} , i.e.

$$m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) + \vec{F} \rightarrow ⑦$$

This external force \vec{F} will cause a ^{uniform} drift of the guiding centre with velocity given by -

$$\vec{v}_F = \frac{\vec{F} \times \vec{B}}{q B^2} \quad (\text{already discussed in the previous topics})$$

→ ⑧

(8)

In our present case, the 2nd term in LHS of eqn(6) constitutes the force term of eqn(7). This addition force, however, is not uniform (constant) over the region. Thus small oscillations occur during one period of gyration. But we will eliminate these small oscillations by averaging this force term over one gyration period so that - the motion of the guiding centre can be smoothed. (i.e. we are interested in the smoothed motion of the guiding centre only).

Now we are going to examine some simplest cases, where -

① $\vec{D}B$ is perpendicular to \vec{B}

and ②. The lines of force are assumed to be curved with a constant radius of curvature.

(7)

Case - I - $\vec{D} \vec{B} \perp \vec{B}$; grad \vec{B} drift

We assume that - the lines of force are straight, but their density increases, say in the dirⁿ of y -axis, while \vec{B} are pointing along z -axis. Thus,

$$\vec{B}_0 \longrightarrow \hat{k} B_{0z}$$

$$\vec{v}_0 \longrightarrow i v_{ox} + j v_{oy}. \quad (\text{ideal gyration about } \vec{B}_0 \text{ on } x-y \text{ plane}).$$

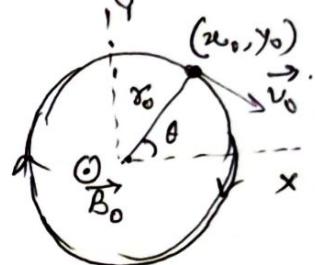
$$\vec{r}_0 \longrightarrow i r_0 \cos \theta + j r_0 \sin \theta.$$

$$= i (r_0 \cos \theta) + j (r_0 \sin \theta)$$

$$(\vec{r}_0 \cdot \vec{D}) \vec{B} \rightarrow \left((r_0 \cos \theta) \frac{\partial}{\partial x} + (r_0 \sin \theta) \frac{\partial}{\partial y} \right) (\hat{k} B_{0z})$$

$$= \hat{k} \left[\underbrace{(r_0 \cos \theta) \frac{\partial B_{0z}}{\partial x}}_{= 0} + (r_0 \sin \theta) \frac{\partial B_{0z}}{\partial y} \right].$$

$$= \hat{k} (r_0 \sin \theta) \frac{\partial B_{0z}}{\partial y}.$$



$$\begin{aligned} \therefore \vec{v}_0 \times (\vec{r}_0 \cdot \vec{D}) \vec{B} &= (i v_{ox} + j v_{oy}) \times \hat{k} (r_0 \sin \theta) \frac{\partial B_{0z}}{\partial y} \\ &= (i v_0 \cos \theta + j v_0 \sin \theta) \times \hat{k} (r_0 \sin \theta) \frac{\partial B_{0z}}{\partial y} \\ &= \left[i v_0 \cos \left(\frac{\pi}{2} - \theta \right) + j v_0 \sin \left(\frac{\pi}{2} - \theta \right) \right] \times \hat{k} (r_0 \sin \theta) \frac{\partial B_{0z}}{\partial y} \\ &= -j v_0 r_0 \sin \theta \frac{\partial B_{0z}}{\partial y} - i v_0 r_0 \sin \theta \cos \theta \frac{\partial B_{0z}}{\partial y}. \end{aligned}$$

Now, these terms are slowly varying f's in the above expression inside the particle orbit, so

averaging each term for a complete rotation -

(10a)

$$\langle \vec{v}_0 \times (\vec{r}_0 \cdot \vec{D}) \vec{B} \rangle = -j v_0 r_0 \left(\frac{\partial B_z}{\partial y} \right).$$

$$\therefore \langle \sin^2 \theta \rangle = \frac{\int_{0}^{2\pi} \sin^2 \theta d\theta}{\int_{0}^{2\pi} d\theta} = \frac{1}{2}$$

and $\langle \sin \theta \cos \theta \rangle = \frac{\int_{0}^{2\pi} \sin \theta \cos \theta d\theta}{\int_{0}^{2\pi} d\theta} = 0.$

$$\begin{aligned} \text{Hence, } \langle \vec{F} \rangle &= \langle q \left\{ \vec{v}_0 \times (\vec{r}_0 \cdot \vec{D}) \vec{B} \right\} \rangle \\ &= F \pm \frac{q}{2} v_0 r_0 \left(j \frac{\partial B_z}{\partial y} \right) \\ &= \mp \frac{q}{2} v_0 r_0 (\vec{D} \cdot \vec{B}) \end{aligned}$$

This form is responsible for producing a drift of the guiding centre with velocity -

$$\begin{aligned} \vec{v}_{DB} &= \mp \frac{q}{2} v_0 r_0 \frac{\vec{D} \cdot \vec{B} \times \vec{B}}{B^2} \quad \text{by (8).} \quad \left| \begin{array}{l} \frac{mv_0^2}{r_0} = qV_0 B \\ \Rightarrow \frac{mv_0}{2B} = r_0 \end{array} \right. \\ &= \pm \frac{q}{2} v_0 r_0 \frac{\vec{B} \times \vec{D} \cdot \vec{B}}{B^2} \\ &= \pm v_0 \left(\frac{mv_0}{qB} \right) \frac{\vec{B} \times \vec{D} \cdot \vec{B}}{B^2} \quad \xrightarrow{\text{(\pm sign goes with the sign of the charge)}} \\ \boxed{\vec{v}_{DB} = \pm \frac{mv_0^2}{2} \frac{\vec{B} \times \vec{D} \cdot \vec{B}}{B^3}} \quad &\longrightarrow (10) \end{aligned}$$

The quantity \vec{v}_{DB} is called grad-B drift, and it is in opposite directions for positive ions and electrons. Therefore grad-B drift causes a net electric current transverse to \vec{B} and $\vec{D} \cdot \vec{B}$. In our case, the guiding centres for the gyrating orbits of ions are drifting in -ve x-dirⁿ, while those for e⁻s are drifting in +x-dirⁿ as shown in fig-2.

\rightarrow **B** OUT OF PAGE

106

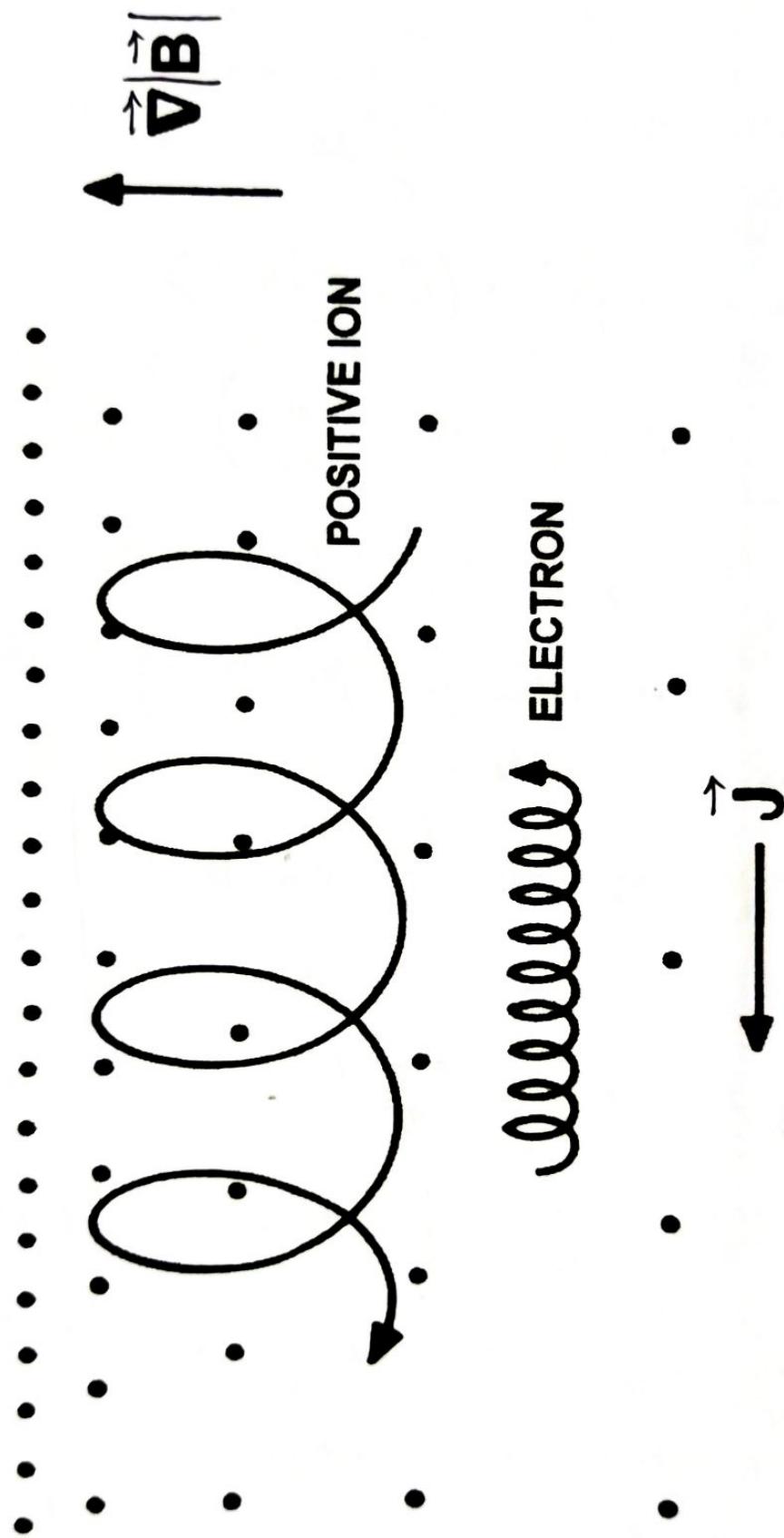


Fig. Charged particle drifts due to a \mathbf{B} field gradient perpendicular to \mathbf{B} .

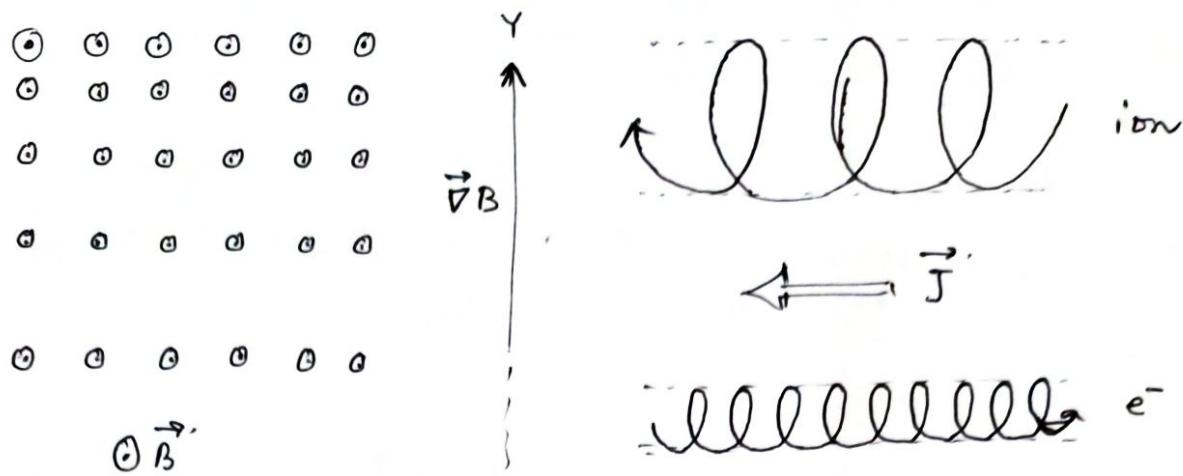


Fig-2 Charged particle drifts due to a \vec{B} field gradient perpendicular to \vec{B} .

The physical reason for drifting of gyring centre can be shown easily. Since De Larmour radius decreases with increase of the concentration of magnetic field (i.e. with increase of the magnetic field strength), the orbit of the charged particles gyrating about \vec{B} will have ^{smaller} radius of curvature in the region of stronger \vec{B} field (so in the upper half of the circular orbit) and in the lower half, the radius of curvature will be larger. So, the ions are gyrating clockwise (as in our diag-2), they drift to the left. For e^- s, they are gyrating anti-clockwise and so, they drift towards right.

Case-II Curved \vec{B} : curvature drift.

In case-I, we have assumed that there is a gradient in the magnetic field strength \vec{B} , but the lines of the magnetic induction \vec{B} field are straight, $\vec{B} \cdot \begin{pmatrix} \vec{z} \\ \vec{y} \end{pmatrix}$. The vector \vec{B} is purely in the z -dirⁿ.

Now we are making another assumption: that the field lines are locally curved with radius of curvature R_c , but the field strength B is locally constant.

A guiding centre drift arises from the centrifugal force experienced by the charged particles as they move along the field lines in their

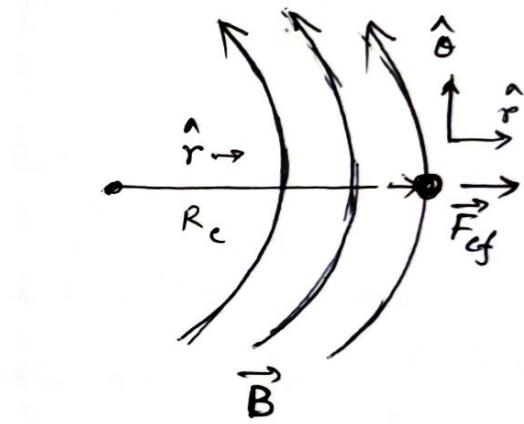
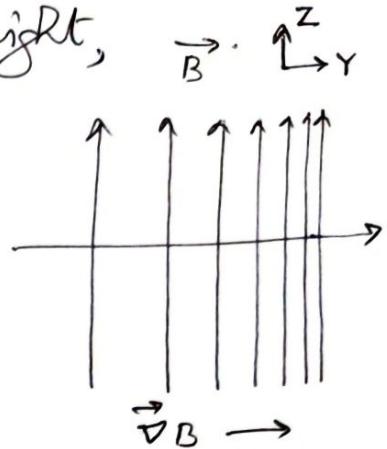


Fig-3

thermal motion. The average centrifugal force acting on the charged particle is given by -

$$\vec{F}_{cf} = \frac{m v_{||}^2}{R_c} \hat{r} = m v_{||}^2 \frac{\vec{R}_c}{R_c^2}$$

where $v_{||}^2$ denotes the average square of the component of random velocity (thermal) along \vec{B} and R_c is the radius of curvature of the field line w.r.t. to its local centre of curvature. Considering this force, the sign of motion becomes -

$$m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) + \vec{F}_{cf}$$

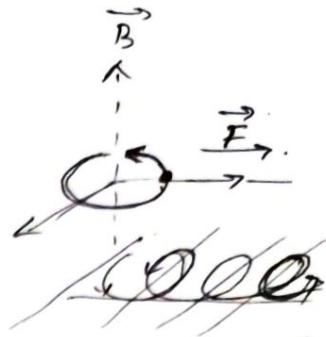
(13)

The \vec{F}_{cf} force gives rise to the drift of guiding centre with the velocity -

$$\vec{v}_R = \pm \frac{\vec{F}_{cf} \times \vec{B}}{B^2}$$

$$= \pm \frac{1}{2} \frac{mv_{||}^2}{R_c^2} \frac{\vec{R}_c \times \vec{B}}{B^2}$$

$$\boxed{\vec{v}_R = \pm \frac{mv_{||}^2}{2B^2} \frac{\vec{R}_c \times \vec{B}}{R_c^2}}$$



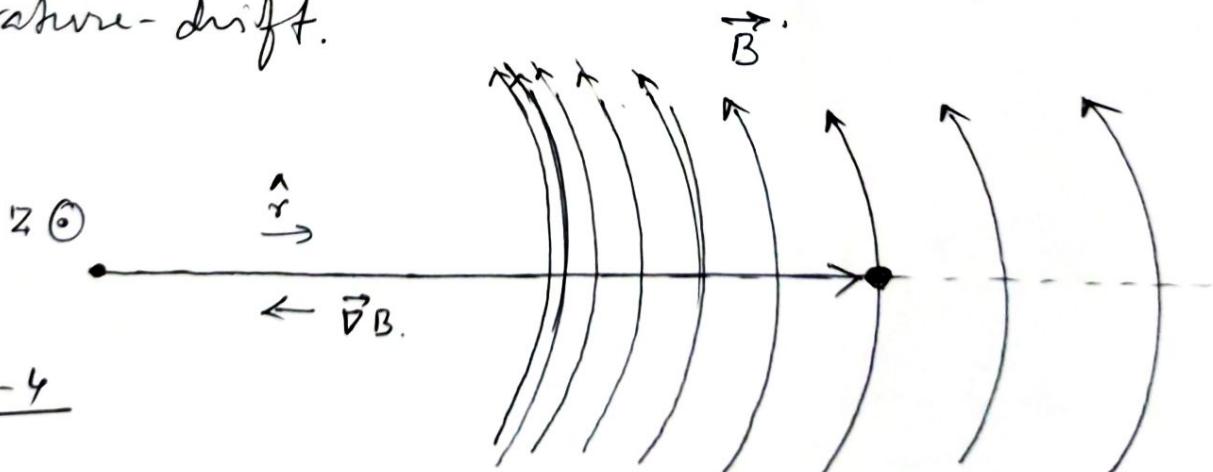
$$\omega_c = \frac{eB}{mv}$$

→ (12).

The guiding centre will drift along the dirⁿ ⊥ to the plane of \vec{R}_c and \vec{B} .

Case-III - Combined gradient-curvature drift. -

Now, we suppose that there is a decrease of $|B|$ with radius. For such a situation, we must compute the grad-B drift, which will now accompany the curvature-drift.



grad-B
curvature-drift

$$\vec{v}_{RB} = \pm \frac{1}{2} \frac{v_{||}^2}{\omega_c} \frac{\vec{B} \times \vec{\nabla} B}{B^2}$$

→ (10).

In vacuum, $\vec{\nabla} \times \vec{B} = 0$ $\left(\vec{J} = 0 \right)$.

$$\Rightarrow \frac{1}{r} \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ B_r & rB_\theta & B_z \end{vmatrix} = 0.$$

$$\vec{\nabla} \times \vec{A} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \hat{e}_1 & h_2 \hat{e}_2 & h_3 \hat{e}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ A_1 h_1 & A_2 h_2 & A_3 h_3 \end{vmatrix}$$

For cylindrical coordinates (r, θ, z)
 $h_1 = 1, h_2 = r, h_3 = 1$

$$\Rightarrow \frac{1}{r} \left[\hat{r} \left\{ 0 - \frac{\partial}{\partial z} (rB_\theta) \right\} + \hat{\theta} (0 - 0) + \hat{z} \left\{ \frac{\partial}{\partial r} (rB_\theta) - 0 \right\} \right] = 0.$$

$$\Rightarrow \frac{1}{r} \left[-\hat{r} \underbrace{\frac{\partial (rB_\theta)}{\partial z}}_{=0} + \hat{z} \frac{\partial (rB_\theta)}{\partial r} \right] = 0.$$

$$\Rightarrow \hat{z} \frac{1}{r} \frac{\partial (rB_\theta)}{\partial r} = 0.$$

$$\Rightarrow rB_\theta = \text{const.} \rightarrow \boxed{B_\theta \propto \frac{1}{r}} \rightarrow (15)$$

The $\vec{\nabla} \times \vec{B}$ has only a z -component, since \vec{B} has only a θ -component $\vec{B}(r, \theta) = \hat{\theta} B_\theta$ ($B_r = 0 \rightarrow$ along \hat{r}).

But $\vec{\nabla} \cdot \vec{B}$ has only r -component.

$$\vec{\nabla} \cdot \vec{B} = \left(\hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta} + \hat{z} \frac{\partial}{\partial z} \right) B.$$

$$\vec{\nabla} = \frac{\hat{e}_1}{h_1} \frac{\partial}{\partial u_1} + \frac{\hat{e}_2}{h_2} \frac{\partial}{\partial u_2} + \frac{\hat{e}_3}{h_3} \frac{\partial}{\partial u_3}$$

$$\boxed{\vec{\nabla} \cdot \vec{B} = \hat{r} \frac{\partial B}{\partial r}} \quad \left(\because \frac{\partial B}{\partial \theta} = 0 = \frac{\partial B}{\partial z} \right) \rightarrow (14)$$

$$\text{By (13)} \rightarrow |\vec{B}| \propto \frac{1}{r} \quad \left(\vec{B} = \hat{\theta} B_\theta \rightarrow |\vec{B}| = B_\theta \right) \quad (15)$$

$$\text{By (14)} \rightarrow \vec{\nabla}|\vec{B}| \propto \hat{r} \frac{\partial}{\partial r} \left(\frac{1}{r} \right).$$

$$\text{or } \vec{\nabla}|\vec{B}| \propto -\hat{r} \left(\frac{1}{r^2} \right).$$

$$\therefore \frac{\vec{\nabla}|\vec{B}|}{|\vec{B}|} = -\frac{\hat{r}}{r^2} \Rightarrow \frac{\vec{\nabla}B}{B} = -\frac{\vec{r}}{r^2} = -\frac{\vec{R}_c}{R_c^2}$$

Using it in (10) →

$$\vec{v}_{DB} = \pm \frac{1}{2} \frac{v_\perp^2}{\omega_c} \frac{\vec{B} \times \left(-B \frac{\vec{R}_c}{R_c^2} \right)}{B^2}.$$

$$= \mp \frac{1}{2} \frac{v_\perp^2}{\omega_c} \frac{1}{B^2} \left[\vec{B} \times \left(B \frac{\vec{R}_c}{R_c^2} \right) \right]$$

$$\therefore \vec{v}_{DB} = \pm \frac{1}{2} \frac{v_\perp^2}{\omega_c B} \frac{\vec{R}_c \times \vec{B}}{R_c^2}$$

Adding this to the curvative drift \vec{v}_R in (12),

we have the total drift in a curved vacuum magnetic field —

$$\vec{v}_T = \vec{v}_R + \vec{v}_{DB}$$

$$= \frac{mv_{II}^2}{qB^2} \frac{\vec{R}_c \times \vec{B}}{R_c^2} + \frac{mv_\perp^2}{2q} \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2}$$

$$= \frac{mv}{qB^2} \frac{\vec{R}_c \times \vec{B}}{R_c^2} \left[v_{II}^2 + \frac{1}{2} v_\perp^2 \right]$$

$$\omega_c = \frac{qB}{m}$$

$$\boxed{\vec{v}_T = \frac{m}{q} \frac{\vec{R}_c \times \vec{B}}{R_c^2 B^2} \left[v_{II}^2 + \frac{1}{2} v_\perp^2 \right]}$$

→ (16).

(16)

Cause of Plasma Instability

From the equation (16), it is seen that the both grad-B drift due to inhomogeneities (spatial) and curvature-drift due to bending of magnetic lines of force (bending of magnetic field) always add together. So, if there is a bending of magnetic field, the charged particles drift out immediately, no matter how one manipulates/adjusts the temp' and magnetic fields.

PART-III

A promising scheme for plasma confinement:

Magnetic mirrors ($\vec{v} \cdot \vec{B} \parallel \vec{B}$)

We consider a magnetic field which is pointed primarily in the dirⁿ of +z-axis and whose magnitude varies in the Z-dirⁿ. Again, we assume that the field is axis symmetric with $B_\theta = 0$ and $\frac{\partial |B|}{\partial \theta} = 0$.

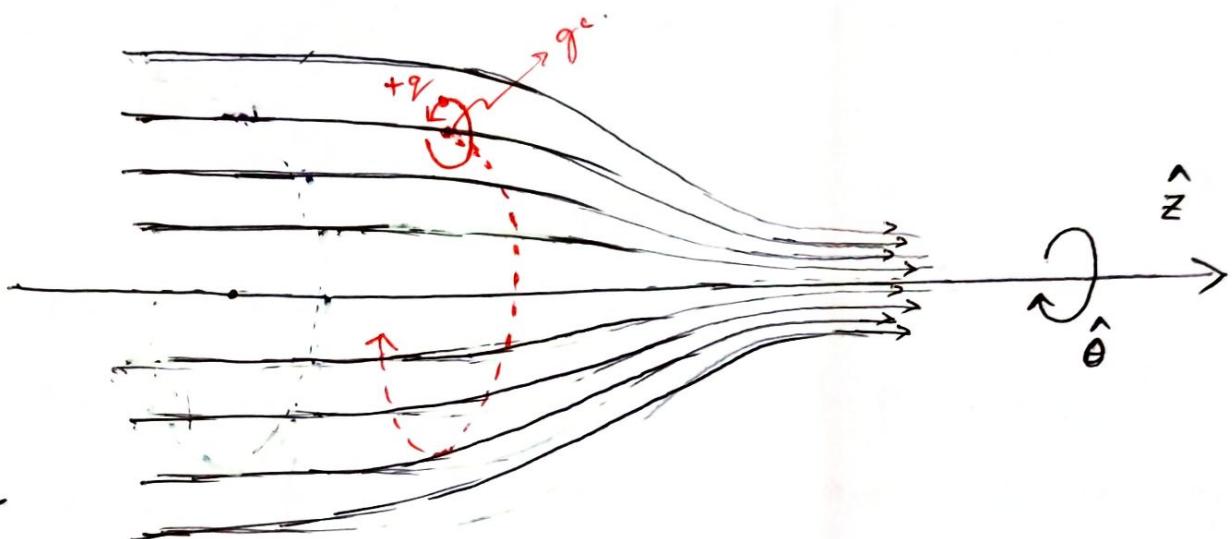


Fig-1

To study the motion of charged particles trapped inside a magnetic field of above character as shown in fig -1, we have to do the following mathematics.

We start with the basic eqn^m of magnetostatics.

$$\begin{aligned}
 & \vec{\nabla}_0 \cdot \vec{B} = 0 \\
 \Rightarrow & \frac{1}{r} \left[\frac{\partial}{\partial r} (B_r \times r) + \frac{\partial}{\partial \theta} (B_\theta) + \frac{\partial}{\partial z} (\alpha B_z) \right] = 0 \\
 \Rightarrow & \frac{1}{r} \left[\frac{\partial}{\partial r} (r B_r) + \frac{\partial B_\theta}{\partial \theta} + \frac{\partial \alpha B_z}{\partial z} \right] = 0 \quad (1) \\
 \Rightarrow & \frac{1}{r} \frac{\partial}{\partial r} (r B_r) + \frac{\partial B_z}{\partial z} = 0. \quad (1)
 \end{aligned}$$

$$\left. \begin{aligned}
 \vec{\nabla}_0 \cdot \vec{A} &= \frac{1}{h_1 h_2 h_3} \left[\frac{\partial (A_1 h_2 h_3)}{\partial u_1} \right. \\
 &\quad \left. + \frac{\partial (A_2 h_3 h_1)}{\partial u_2} + \frac{\partial (A_3 h_1 h_2)}{\partial u_3} \right] \\
 \text{In cylindrical coordinates } &(r, \theta, z) \\
 h_1 &= 1, \quad h_2 = r, \quad h_3 = 1 \\
 A_1 &= A_r, \quad A_2 = A_\theta, \quad A_3 = A_z
 \end{aligned} \right\}$$

(2)

$$\therefore rB_r = - \int_0^r \left(r \frac{\partial B_z}{\partial z} \right) dr$$

$$= - \left(\frac{\partial B_z}{\partial z} \right)_{r=0} \int_0^r r dr.$$

$$\Rightarrow \boxed{B_r = - \frac{r}{2} \left(\frac{\partial B_z}{\partial z} \right)_{r=0}} \rightarrow (2).$$

= a constant value

Supposing $\frac{\partial B_z}{\partial z}$ doesn't vary much with r and the value of the derivative is taken at $r=0$ (on the axis).

Thus, $\vec{B} = \hat{r} B_r(r) + \hat{z} B_z$ ($B_\theta = 0$), at a point $P(r, \theta, z)$ inside the magnetic field.

If we consider a plane $\perp r$ to z -axis, $|\vec{B}|$ is found changing with r carrying a grad- B drift of the guiding centres of the charged particle gyro-orbits about the axis of symmetry. [Since $\vec{v}_{DB} = \pm \frac{1}{2} v_0 r_0 \frac{\vec{B} \times \vec{\nabla} B}{B^2}$,] $\vec{\nabla} B \perp \hat{z}$ and $\vec{\nabla} B \parallel \hat{r}$ on the plane imagined.], but there is no radial drift, seems $\frac{\partial |\vec{B}|}{\partial \theta} = 0$.

Now applying the eqns of motion of a charged particle -

$$\vec{F} = \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) \quad (\text{F= Lorentz force})$$

$$= q \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{z} \\ v_r & v_\theta & v_z \\ B_r & B_\theta & B_z \end{vmatrix}$$

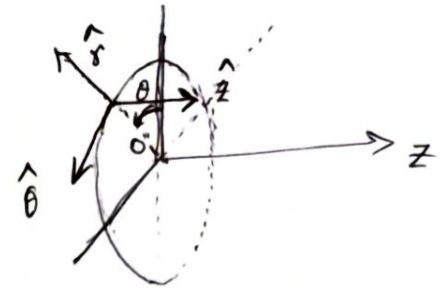
$$= q \hat{r} (v_\theta B_z - v_z B_\theta) + \hat{\theta} (v_z B_r - v_r B_z) + \hat{z} (v_r B_\theta - v_\theta B_r)$$

v_0

(3)

Here, the components of \vec{F} along \hat{r} , $\hat{\theta}$ and \hat{z} are.

$$* \begin{bmatrix} \hat{r} & \rightarrow \text{increasing } r \\ \hat{\theta} & \rightarrow " \theta \\ \hat{z} & \rightarrow " z. \end{bmatrix}$$



$$\left. \begin{aligned} F_r &= q v_0 B_z \\ F_\theta &= q(v_z B_r - v_r B_z) \\ F_z &= -q v_0 B_r. \end{aligned} \right\} (3)$$

$$\left. \begin{aligned} \vec{B} &= \hat{r} B_r + \hat{z} B_z. \\ \text{At the axis} & \vec{B} \parallel \hat{z} \rightarrow B_r = 0. \\ \vec{v}_F &= \frac{\vec{F} \times \vec{B}}{q B^2}, F \rightarrow F_F \end{aligned} \right\}$$

The combination of F_r and F_θ causes Larmor gyration about the magnetic field and also causes

a drift in radial dirⁿ on the plane \perp to z-axis. Again, the F_z component acting along z-axis makes the guiding centres to follow the lines of force.

Now, concentrating on the force component F_z .

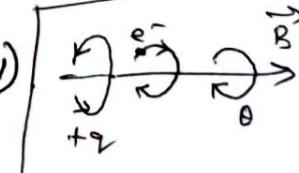
$$F_z = \frac{1}{2} q v_0 r \left(\frac{\partial B_z}{\partial z} \right) \rightarrow (4)$$

Now we will calculate F_z averaging over one gyration. For simplicity, we consider the a gyrating particle whose guiding centre lies on the z-axis.

$$v_0 \rightarrow v_L, r \rightarrow r_L \text{ and } \omega_c = \frac{qB}{m} \rightarrow \omega_c r_L = v_L$$

$$\therefore \langle F_z \rangle = \frac{1}{2} q v_L r_L \left(\frac{\partial B_z}{\partial z} \right) = \frac{1}{2} q \frac{v_L}{\omega_c} \left(\frac{\partial B_z}{\partial z} \right) = \frac{1}{2} \frac{mv_L^2}{B} \left(\frac{\partial B_z}{\partial z} \right) \rightarrow (5).$$

Depending on the sign of q , v_0 is $\pm v_L$ (fig - 1)



(4)

Introducing the magnetic moment of gyrating charge

particle -

$$\boxed{\mu = \frac{\frac{1}{2}mv_L^2}{B}} \rightarrow \textcircled{4}$$

in the above eqn $\textcircled{5}$ \rightarrow

$$\langle F_z \rangle = -\mu \frac{\partial B_z}{\partial z} \rightarrow \textcircled{5}$$

$$\text{So } \vec{F} = \underbrace{\hat{r} f_r + \hat{\theta} f_\theta}_{= \vec{F}_\perp} + \hat{z} f_z$$

$$\left. \begin{aligned} \mu &= I \times A \\ &= \frac{q}{T} \times (\pi r_L^2) \\ &= \frac{q}{2\pi/\omega_c} \times \pi r_L^2 \\ &= \frac{1}{2} q \omega_c r_L^2 \\ &= \frac{1}{2} q \frac{v_L^2}{\omega_c} \\ &= \frac{1}{2} q \frac{v_L^2}{(2B/m)} \end{aligned} \right\}$$

$$\therefore |\vec{F}_\parallel| = m \frac{dv_{\parallel}}{dt} = -\mu \frac{\partial B_z}{\partial z} \rightarrow \text{general } \boxed{F_\parallel = m \frac{dv_{\parallel}}{dt} = -\mu \frac{\partial B}{\partial z}} \rightarrow \textcircled{6}$$

Multiplying both sides by v_{\parallel} $\text{of } \textcircled{6}$ \rightarrow

when $d\vec{s}$ is parallel to \vec{B} .

$$\begin{aligned} m v_{\parallel} \frac{dv_{\parallel}}{dt} &= -\mu \frac{\partial B}{\partial z} v_{\parallel} \\ \Rightarrow \frac{d}{dt} \left(\frac{1}{2} m v_{\parallel}^2 \right) &= -\mu \frac{\partial B}{\partial z} \left(\frac{d\vec{s}}{dt} \right) \left(\vec{v}_{\parallel} = \frac{d\vec{s}}{dt} \right) \\ &= -\mu \frac{dB}{dt}. \quad \left(\frac{dB}{dt} \right. \text{ is the variation of } B \text{ as } \\ &\quad \left. \text{seen by the moving particle.} \right) \end{aligned}$$

$$\Rightarrow \frac{d}{dt} \left(\frac{1}{2} m v_{\parallel}^2 + \mu B \right) = B \frac{d\mu}{dt}$$

$$\Rightarrow \frac{d}{dt} \left(\frac{1}{2} m v_{\parallel}^2 + \frac{1}{2} m v_L^2 \right) = B \frac{d\mu}{dt}$$

$$= \frac{1}{2} m v_0^2 = E \rightarrow \text{total KE. = const.}$$

$$\Rightarrow \boxed{\frac{d\mu}{dt} = 0}, \quad B \neq 0. \rightarrow \textcircled{6}$$

i.e. as the particle moves into a stronger or weaker magnetic field B , its Larmor radius changes ($\because r_L = \frac{mv_L}{qB}$), but μ remains invariant. The total energy $E = \frac{1}{2} m v_{\parallel}^2 + \frac{1}{2} m v_L^2 = \frac{1}{2} m v_{\parallel}^2 + \mu B = \text{const.} \rightarrow \textcircled{7}$.

Now, the question is what is the inside physics of μ (magnetic moment) invariance for the charged particles confined in a magnetic field with $\vec{v} \parallel \vec{B}$.

As the charged particle moves from a weak-field region to a strong-field region (from left to right as in fig-1) in course of its thermal motion, it sees an increasing B , and therefore its v_{\perp} must increase in order to keep μ constant. (by eqn-④). Since its total energy ($\frac{1}{2}mv_{\parallel}^2 + \frac{1}{2}mv_{\perp}^2$) must remain constant, v_{\parallel} must necessarily decrease. (The same result can again be obtained from eqn-⑦. With increase of B , v_{\parallel} must decrease, since total energy and μ are both constants.) If B is high enough in the 'throat' of the 'magnetic bottle', v_{\parallel} eventually becomes zero; and the particle has no option other than turning (reflecting) back to the weak-field region. So, the 'throat' of the magnetic bottle where the field strength is very high behaves as a 'mirror' causing reflection of the charged particle coming to that region. This effect works on both ions and electrons. So, the charged particles (plasma) can be trapped in a nonuniform magnetic field formed between

(6)

two magnetic mirrors (Fig-2).

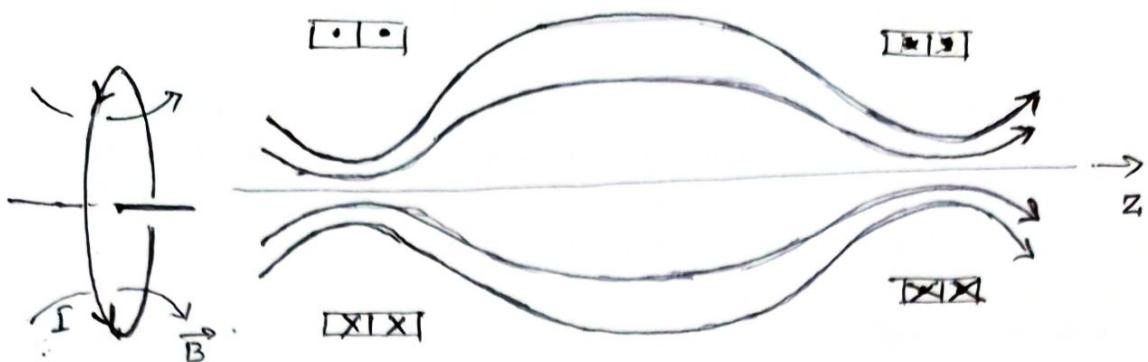


Fig-2 Non-uniform magnetic field formed between two magnetic mirrors. The field is formed by two pair of current carrying coils.

The whole physics follows from the basic fact that magnetic moment μ for the charged particle is an invariant and invariance of μ is the basis for one of the primary schemes for plasma confinement:

Magnetic mirror (Magnetic bottle).

* Egmr (6): i.e. constancy of the charged particle magnetic moment holds only with the approximation and, i.e. when the spatial variation of \vec{B} inside the gyro-orbit is very much small compared to the magnitude of \vec{B} ($\delta B = \frac{r}{c} |\nabla B| \ll B$). That is why, the orbital magnetic moment of the charge particle, i.e. μ is said to be an adiabatic invariant. It is usually referred to as the 1st adiabatic invariant in Plasma Physics.



Magnetic Mirror Effect

μ

As a consequence of the adiabatic invariance of $|m|$ and Φ_m , as the particle moves into a region of converging magnetic field lines its transverse kinetic energy W_{\perp} increases, while its parallel kinetic energy W_{\parallel} decreases, in order to keep $|m|$ and the total energy constant. Ultimately, if the \mathbf{B} field becomes strong enough, the particle velocity in the direction of increasing field may eventually come to zero and then be reversed. After reversion, the particle is speeded up in the direction of decreasing field, while its transverse velocity diminishes. Thus, the particle is *reflected* from the region of converging magnetic field lines. This phenomenon is called the *magnetic mirror effect* and is the basis for one of the primary schemes of plasma confinement.

When two coaxial magnetic mirrors are considered, as illustrated in Fig. 3, the charged particles may be reflected by the magnetic mirrors and may travel back and forth in the space between them, becoming trapped. This trapping region has been called a *magnetic bottle* and it has been used in laboratory for plasma confinement.

The trapping in a magnetic mirror system is not perfect, however. The effectiveness of a coaxial magnetic mirror system in the trapping of charged particles can be measured by the mirror ratio B_m/B_0 , where B_m

Why?

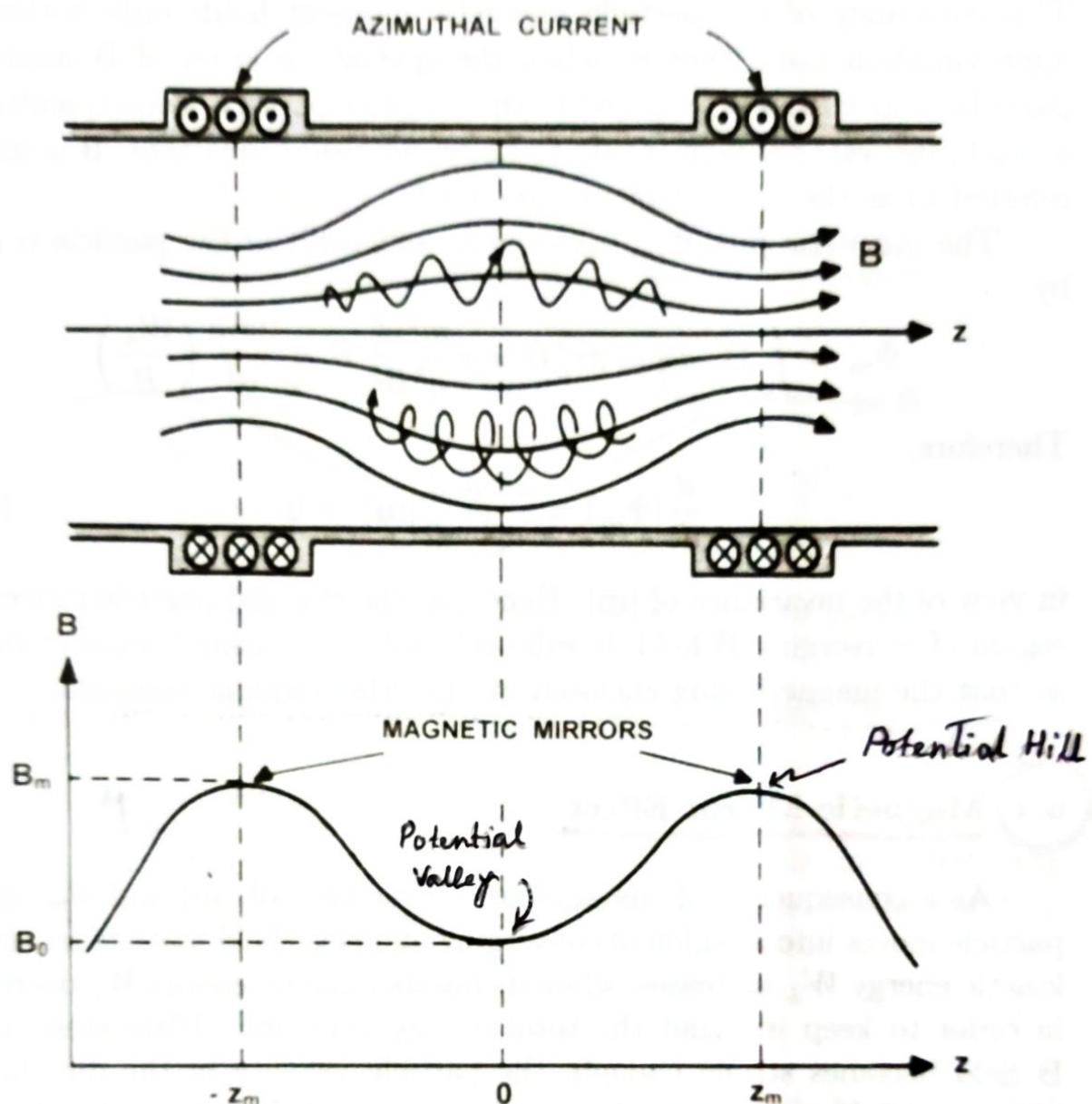
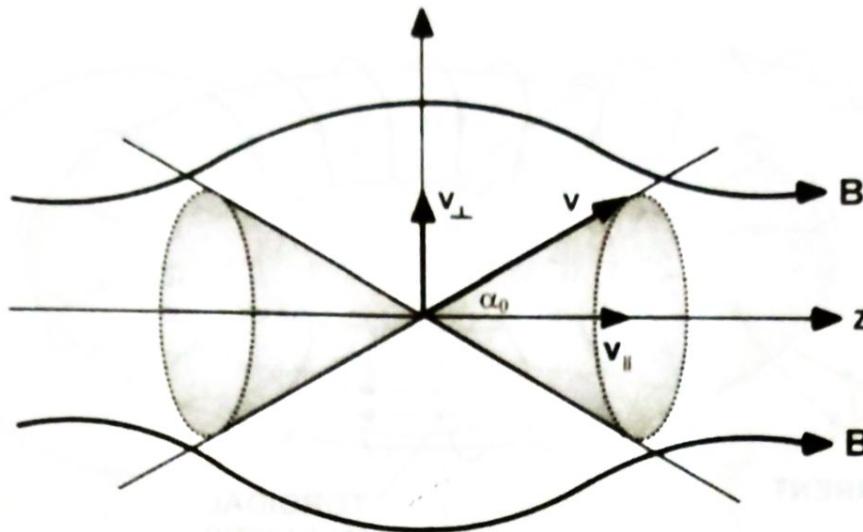


Fig.3 Schematic diagram showing the arrangement of coils to produce two coaxial magnetic mirrors facing each other, for plasma confinement, and the relative intensity variation of the magnetic field.

is the intensity of the magnetic field at the point of reflection (where the pitch angle of the particle is $\pi/2$) and B_0 is the intensity of the magnetic field at the center of the magnetic bottle.

Consider a charged particle having a pitch angle α_0 at the center of the magnetic bottle. If v is the particle speed, which in a static magnetic field

(7)



$$v_{||} = v \cos \alpha$$

$$v_{\perp} = v \sin \alpha$$

Fig. 12 The loss cone in a coaxial magnetic mirror system.

remains constant, the constancy of the magnetic moment $\underline{m} = W_{\perp}/B = \frac{\frac{1}{2}mv_{\perp}^2}{B}$ leads to

$$\underline{m}_{z=z} = \underline{m}_{z=0} \Rightarrow \frac{1}{2}mv^2(\sin^2 \alpha)/B = \frac{1}{2}mv^2(\sin^2 \alpha_0)/B_0 \text{ at center (6.11)}$$

where α is the particle pitch angle at a position where the magnetic field intensity is B . Thus, at any point inside the magnetic bottle, for this particle,

$$\frac{\sin^2 \alpha(z)}{B(z)} = \frac{\sin^2 \alpha_0}{B_0} \quad (6.12)$$

$$\begin{cases} B(z=z) = B \\ B(z=0) = B_0 \\ B(z=z_m) = B_m \end{cases}$$

↑
Eq - 11

Suppose now that this particle is reflected at the *throat* of the mirror, that is, $\alpha = \pi/2$ for $B(z) = B_m$. Therefore, from (6.12),

$$v_{||} \rightarrow 0, v \rightarrow v_{\perp} \quad z = \pm z_m \quad (\sin^2 \alpha_0)/B_0 = 1/B_m \quad (6.13)$$

This means that a particle having a pitch angle α_0 given by

$$\alpha_0 = \sin^{-1}[(B_0/B_m)^{1/2}] = \sin^{-1}(v_{\perp}/v)_0 \quad (6.14)$$

at the center of the bottle, is reflected at a point where the intensity of the field is B_m . Therefore, for a magnetic bottle with a fixed mirror ratio B_m/B_0 , the plasma particles having a pitch angle at the center greater than α_0 , as given by (6.14), will be reflected before the ends of the magnetic bottle. On the other hand, if the pitch angle of the particle at the center is

$$\underline{m}(z=0) = \underline{m}(z=\pm z_m) \Rightarrow \frac{\frac{1}{2}mv_{\perp}^2}{B_0} = \frac{\frac{1}{2}mv_0^2}{B_m} \Rightarrow \boxed{\frac{B_0}{B_m} = \frac{v_{\perp}^2}{v_0^2}}$$

$$v_0 = v_{\perp}$$

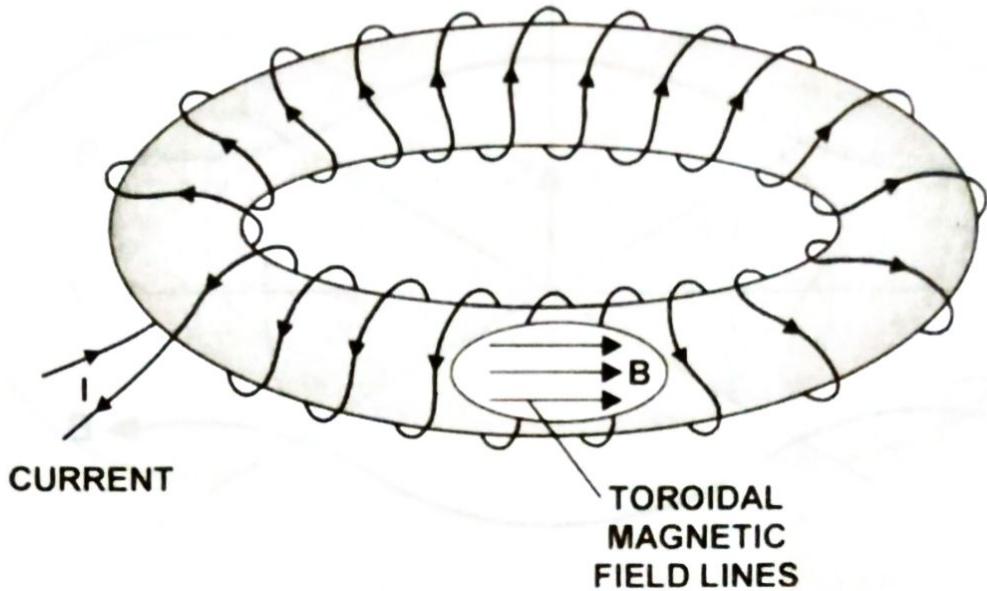


Fig. 13 Magnetic field with toroidal geometry.

less than α_0 , its pitch angle will never reach the value $\pi/2$, which implies that at the ends of the bottle the particle has a non-vanishing parallel velocity and hence escapes through the ends of the mirror system. There is, therefore, a loss cone, a cone of half-angle α_0 with its vertex at the center, as shown in Fig. 12, where particles that have velocity vectors with a pitch angle falling inside it are not trapped. The loss cone is determined by the mirror ratio B_m/B_0 , according to (6.14).

Devices that have no ends, with geometries such that the magnetic field lines close on themselves, offer many advantages for plasma confinement. Toroidal geometries (Fig. 13), for example, have no ends, but it turns out that confinement of a plasma inside a toroidal magnetic field does not provide a plasma equilibrium situation, because of the radial inhomogeneity of the field. In this case a poloidal magnetic field is normally superposed on the toroidal field, resulting in *helical* field lines (as in the *Tokamak*). The major problem in most plasma confinement schemes, however, is that instabilities and small fluctuations from the desired equilibrium configuration are always present, which lead to a rapid escape of the particles from the magnetic bottle. This instability problem is a fundamental one, and it is likely to occur in any conceivable magnetic confinement scheme.

A good example of a natural magnetic bottle is the Earth's magnetic field, which traps charged particles of solar and cosmic origin. These

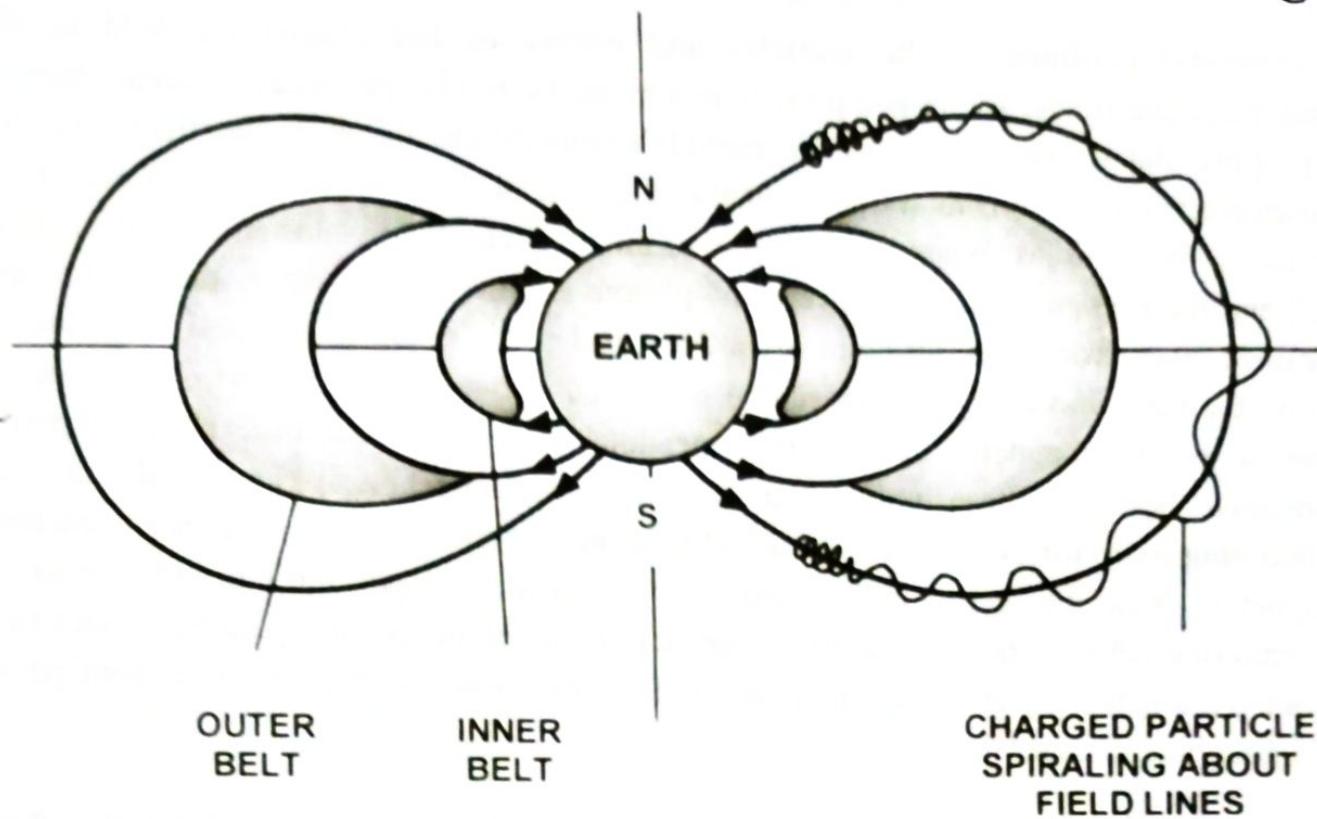


Fig. 14 Dipole approximation of the Earth's magnetic field. The distance of the Van Allen radiation belts from the center of the Earth, at the equator, is about 1.5 Earth radii for the high-energy protons and about 3 to 4 Earth radii for the high-energy electrons.

charged particles trapped in the Earth's magnetic field constitute the so-called *Van Allen radiation belts*. As shown in Fig. 14, the geomagnetic field near the Earth is approximately that of a dipole, with the field lines converging towards the north and south magnetic poles.

The electrons and protons that are trapped in the Van Allen radiation belts spiral in almost helical paths along the field lines, and towards the magnetic poles, where they are eventually reflected. These particles bounce back and forth between the poles. In addition to this bouncing motion, these trapped charged particles are also subject to a gradient drift and a curvature drift in the east-west direction, to be discussed later in this chapter.

Magnetic confinement (Source- internet)

In magnetic confinement the particles and energy of hot plasma are held in place using magnetic fields. A charged particle in a magnetic field experiences a Lorentz force that is proportional to the product of the particle's velocity and the magnetic field. This force causes electrons and ions to spiral about the direction of the magnetic line of force, thereby confining the particles. When the topology of the magnetic field yields an effective magnetic well and the pressure balance between the plasma and the field is stable, the plasma can be confined away from material boundaries. Heat and particles are transported both along and across the field, but energy losses can be prevented in two ways. The first is to increase the strength of the magnetic field at two locations along the field line. Charged particles contained between these points can be made to reflect back and forth, an effect called **magnetic mirroring**. In a basically straight system with a region of intensified magnetic field at each end, particles can still escape through the ends due to scattering between particles as they approach the mirroring points. Such end losses can be avoided altogether by creating a magnetic field in the topology of a torus (i.e., configuration of a doughnut or inner tube).

External magnets can be arranged to create a magnetic field topology for stable plasma confinement, or they can be used in conjunction with magnetic fields generated by currents induced to flow in the plasma itself. The late 1960s witnessed a major advance by the Soviet Union in harnessing fusion reactions for practical energy production. Soviet scientists achieved a high plasma temperature (about 3,000,000 K), along with other physical parameters, in a machine referred to as a tokamak.

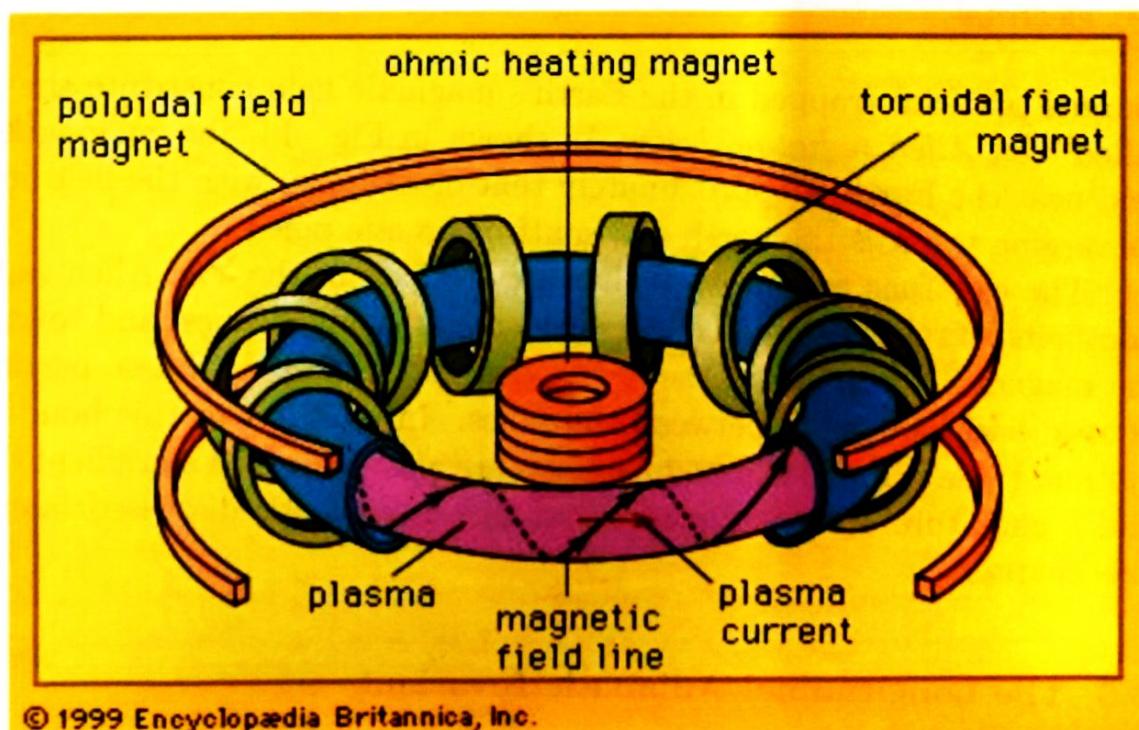
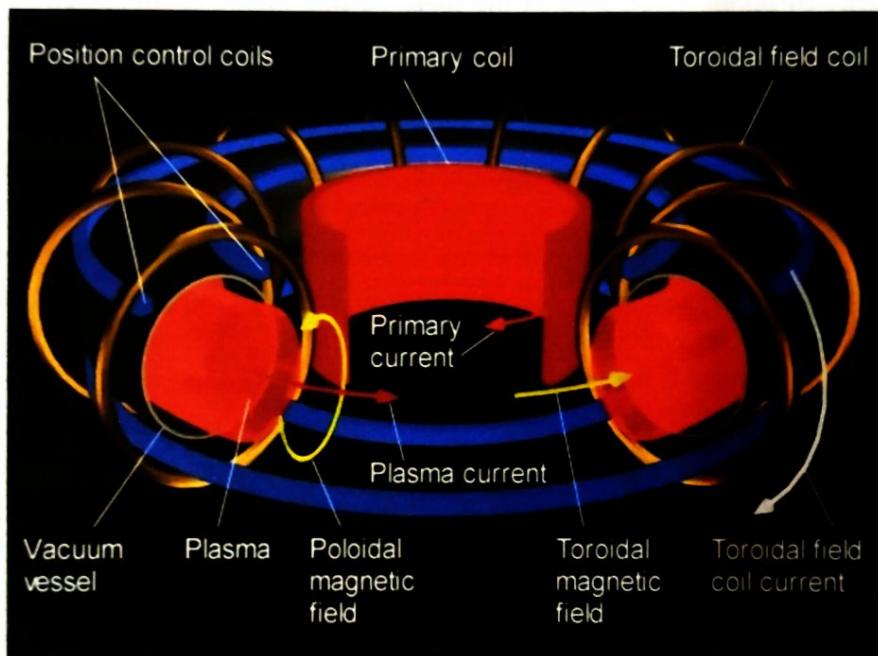


Fig:-Tokamak magnetic confinement.

A tokamak is a toroidal magnetic confinement system in which the plasma is kept stable both by an externally generated, doughnut-shaped magnetic field and by electric currents flowing within the plasma. Since the late 1960s the tokamak has been the major focus of magnetic fusion research worldwide, though other approaches such as the stellarator, the compact torus, and the reversed field pinch (RFP) have also been pursued. In these approaches, the magnetic field lines follow a helical, or screwlike, path as the lines of magnetic force proceed around the torus. In the tokamak the pitch of the helix is weak, so the field lines wind loosely around the poloidal direction (through the central hole) of the torus. In contrast, RFP field lines wind much tighter, wrapping many times in the poloidal direction before completing one loop in the toroidal direction (around the central hole).



Magnetically confined plasma must be heated to temperatures at which nuclear fusion is vigorous, typically greater than 75,000,000 K (equivalent to an energy of 4,400 eV). This can be achieved by coupling radio-frequency waves or microwaves to the plasma particles, by injecting energetic beams of neutral atoms that become ionized and heat the plasma, by magnetically compressing the plasma, or by the ohmic heating (also known as Joule heating) that occurs when an electric current passes through the plasma.

Employing the tokamak concept, scientists and engineers in the United States, Europe, and Japan began in the mid-1980s to use large experimental tokamak devices to attain conditions of temperature, density, and energy confinement that now match those necessary for practical fusion power generation. The machines employed to achieve these results include the **Joint European Torus (JET)** of the European Union, the **Japanese Tokamak-60 (JT-60)**, and, until 1997, the **Tokamak Fusion Test Reactor (TFTR)** in the United States. Indeed, in both the TFTR and the JET devices, experiments using deuterium and tritium produced more than 10 megawatts of fusion power and essentially energy breakeven

conditions in the plasma itself. Plasma conditions approaching those achieved in tokamaks were also achieved in large **stellarator** machines in Germany and Japan during the 1990s.

Inertial confinement fusion (ICF)

In this approach, a fuel mass is compressed rapidly to densities 1,000 to 10,000 times greater than normal by generating a pressure as high as 10^{17} pascals (10^{12} atmospheres) for periods as short as a nanosecond (10^{-9} second). Near the end of this time period, the implosion speed exceeds about 3×10^5 metres per second. At maximum compression of the fuel, which is now in a cool plasma state, the energy in converging shock waves is sufficient to heat the very centre of the fuel to temperatures high enough to induce fusion reactions (greater than an equivalent energy of about 4,400 eV). If the mass of this highly compressed fuel material is large enough, energy will be generated through fusion reactions before this hot plasma ball disassembles. Under proper conditions, much more energy can be released than is required to compress and shock heat the fuel to thermonuclear burning conditions.

The physical processes in ICF bear a relationship to those in thermonuclear weapons and in star formation—namely, collapse, compression heating, and the onset of nuclear fusion. The situation in star formation differs in one respect: gravity is the cause of the collapse, and a collapsed star begins to expand again due to heat from exoergic nuclear fusion reactions. The expansion is ultimately arrested by the gravitational force associated with the enormous mass of the star, at which point a state of equilibrium in both size and temperature is achieved. In contrast, the fuel in a thermonuclear weapon or ICF completely disassembles. In the ideal ICF case, however, this does not occur until about 30 percent of the fusion fuel has burned.

Over the decades, very significant progress has been made in developing the technology and systems for high-energy, short-time-pulse drivers that are necessary to implode the fusion fuel. The most common driver is a high-power laser, though particle accelerators capable of producing beams of high-energy ions are also used. Lasers that produce more than 100,000 joules in pulses of about one nanosecond are now used in experiments, and the power available in short bursts exceeds 10^{14} watts.

Two lasers capable of delivering up to 5,000,000 joules in equally short bursts, generating a power level on the fusion targets in excess of 5×10^{14} watts, are operational. One facility is the Laser MegaJoule in Bordeaux, France. The other is the National Ignition Facility at the Lawrence Livermore National Laboratory in Livermore, Calif., U.S.



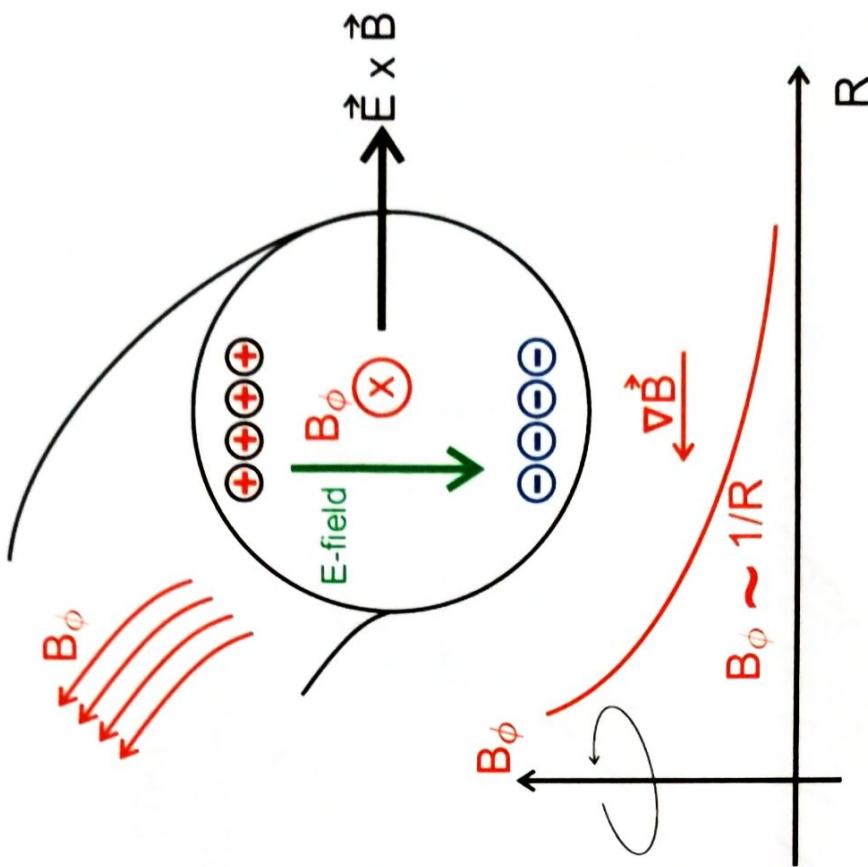
Is a simple toroidal confinement sufficient?

Drift of particles due to forces:

$$\vec{v}_D = \frac{\vec{F} \times \vec{B}}{qB^2} \Rightarrow$$

∇B and curvature drift ("torus drift"):

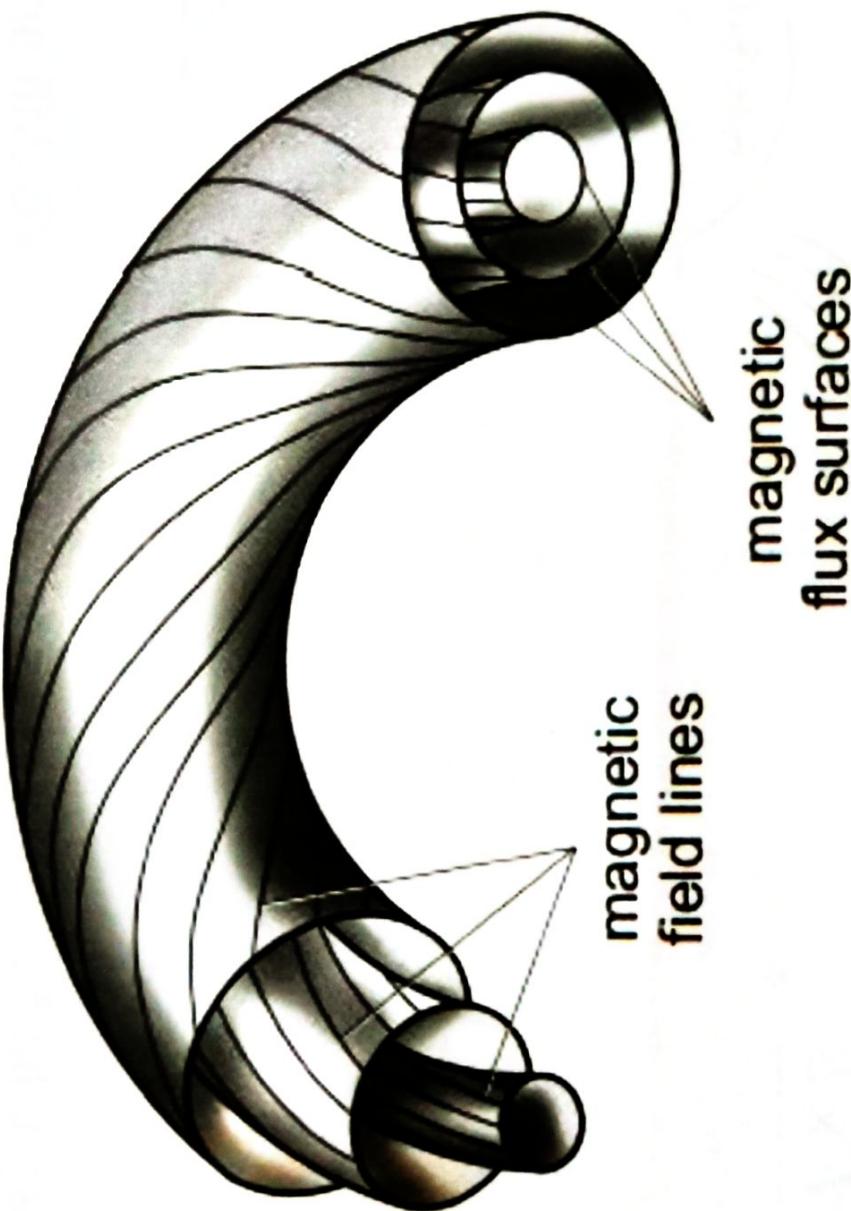
$$\vec{v}_D = \frac{m}{qB^3} \left(v_{\parallel}^2 + \frac{1}{2} v_{\perp}^2 \right) \vec{B} \times \nabla B$$



Pure toroidal magnetic field \rightarrow charge separation via ∇B drift and curvature drift lead to a vertical E-field.

$E \times B$ drift would drive all charged particles towards the outboard side
 \rightarrow Need helical magnetic field structure

Plasma can be confined in a magnetic field



Toroidal systems avoid end losses along magnetic field
→ Need to twist field lines helically to compensate particle drifts

Concepts of magnetic confinement



Tokamak (axis symmetric)



Magnetic field by external coils
and plasma current
pulsed

Stellarator (3D)



Magnetic field only by external coils

intrinsically stationary